DME

//→ Design types:-

- Industrial Design → Product design, Toy & Game design, furniture & interior design, Transport & Mobility design, Ceramic design.
- Textile Design → Apparel design, Accessories.
- Communicatⁿ Design → Animatⁿ, Exhibitⁿ, TV & film, Graphic, Interactⁿ.

//→ Design:-

- (Defⁿ) Design is essentially a decisⁿ making process. If we've a problem, we need to design a solutⁿ. In other words, to design is to formulate a plan to satisfy a particular need & to create something with a physical ~~body~~ reality.
- Engg. design can be defined as the process of applying various techniques to scientific principles for the ~~[illegible]~~ purpose of defining a device, process, or a system in sufficient detail to permit its realizatⁿ.

//→ Simple Machines:-

- Inclined plane
- Screw
- Pulley
- Lever
- Wedge
- Wheel & Axle

//⇒ $$\boxed{\text{Mech Adv.} = \frac{\text{Load}}{\text{Effort}}}$$

- M/c is a device for transferring & transforming motⁿ & power from the source to the load.

- We define a M/C as a combinatⁿ of resisting bodies with successfully constrained relative motⁿs which is used to transform other forms of energy into mech. energy or transmit & modify available energy to do some useful work.

// -> Factors to be considered in M/C design:-

- Mechanism
- Material
- Likely forces
- Size, shape, & space requirements. The final wt. is also a major concern.
- Method of manufacturing.
- How it'll operate?
- Inspectibility
- Reliability & safety aspects.
- Maintainance, cost, & asthetics of the designed product.

// -> Before designing anything, we need data on material, analysis, & user env.

// -> Factor Of Safety (FOS) ->

(IMP) [For ductile materials, FOS is less compared to brittle material. For brittle materials, F.O.S is 2X of ductile materials.]

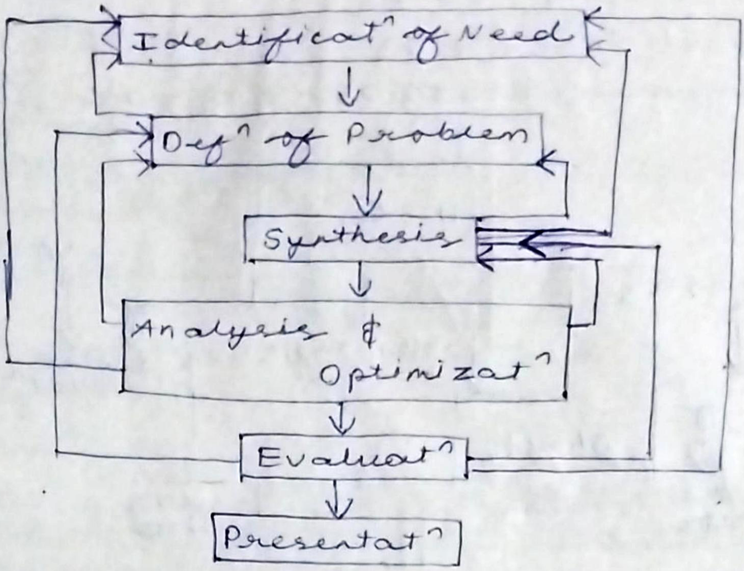
// -> Material Code ->

- Indian Code (BIS)
- German code.
- American Code.
- British code.

// -> Material properties ->

- Yield strength.
- Proof strength.
- Ultimate tensile strength.
- Breaking strength.
- Ductility & Brittleness.
- Maellability.
- Hardness.
- Hardenability.
- Weldability.
- Formability.
- Castability.

// → Designing Process :-



// → Standard :-

(Def^n) A std. is a set of specificat^ns for parts, materials, or processes intended to achieve uniformity, efficiency, & a specified quality. 1 of the imp. purposes of a std. is to limit the multitude of various variat^ns that can arise from the arbitrary creat^n of a part, material, or process.

// → SOP (Std. Operating Procedure or Protocol)

// → Code :-

(Def^n) It's a set of specificat^ns for the analysis, design, manufacture & construct^n of something.

// → Preferred No. :-

It's also called Renard Series.

- $R5 = \sqrt[5]{10} \simeq 1.58$
- $R10 = \sqrt[10]{10} \simeq 1.26$
- $R20 = \sqrt[20]{10} \simeq 1.12$
- $R40 = \sqrt[40]{10} \simeq 1.06$

(a) R10, R20, & R40 : Thickness of sheet metals, wire dia.

(b) R5, R10, & R20 : Speed layout in a m/c tool.

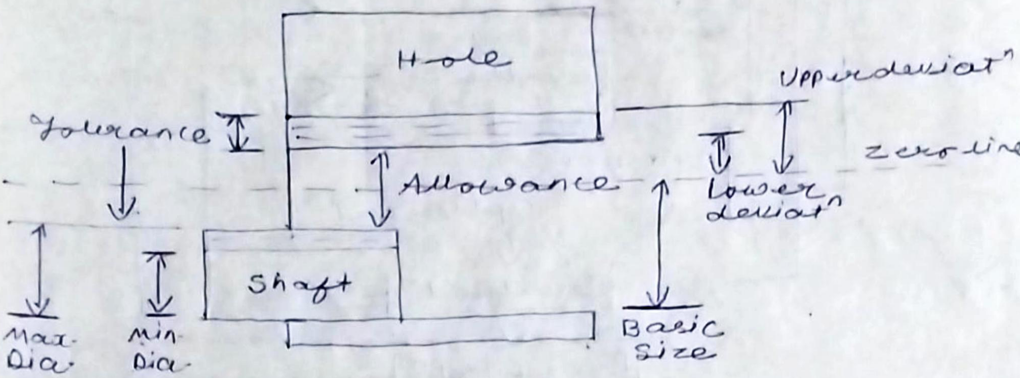
(c) R20 or R40 : M/C tool feed.

(d) R5 : capacities & hydraulic cylinder.

// → Fits & Tolerances :-

Ex: 10 ± 0.05

- Highest limit = 10.05
- Lowest limit = 9.95
- Tolerance limit = 10.05 - 9.95 = 0.1

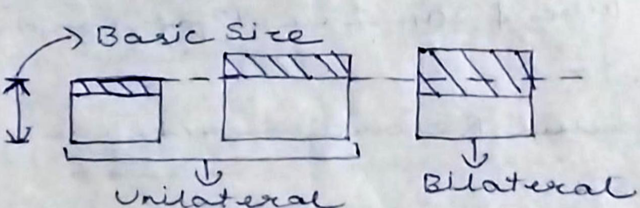


// → Allowance :-

(Defⁿ) [The difference between the dimensⁿs of 2 mating parts is called allowance]

- [Upper deviatⁿ is the difference in dimensⁿ between the max possible size of the component & it's ~~normal~~ nominal size].
- [Lower deviatⁿ is the difference in dimensⁿ between the min. possible size of the component & its nominal value]

// → Tolerance :-



- Ex: $50^{+0.00}_{-0.05}$
- Ex: $50^{+0.05}_{-0.00}$
- Ex: $50^{\pm 0.05}$

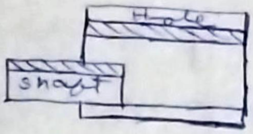
// → Fundamental deviatⁿ :-

(Defⁿ) [It defines the locatⁿ of the tolerance zone w.r.t the nominal size. For this, either of the deviatⁿs may be considered]

// → Fit System :-

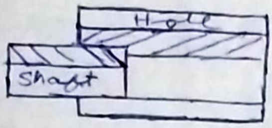
There're 3 types of fit :

(a) Clearance fit



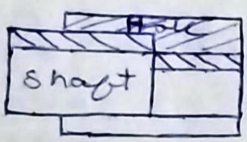
In clearance fit, max. shaft size is less than min. hole size. (Defⁿ)

(b) Transitⁿ fit



In transitⁿ fit, min. hole size is more than min. shaft size, & max. shaft size is less than max. hole size. (Defⁿ)

(c) Interference fit



In interference fit, min. ~~hole~~ shaft size is more than max. hole size. (Defⁿ)

// → Interchangability :-

(Defⁿ) Due to tolerance & stds followed in manufacturing, the mating parts can be easily replaced & this is called interchangability.

// → Member loading :-

There're 2 types of loading i.e. Static & Dynamic loading, & the members have to withstand it.

Loading on m/c elements may be due to.

(a) The energy transmitted by a m/c element.

(b) Dead wt. (c) Inertial forces (d) Thermal loading (e) Frictⁿal forces.

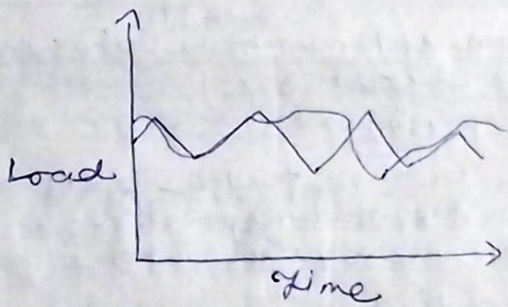
The static load doesn't change in magnitude & directⁿ & normally increases gradually to a steady value.

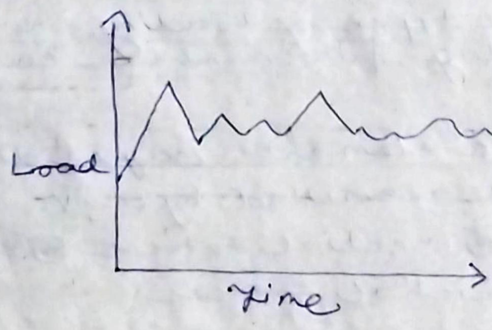
The dynamic load may change in magnitude & directⁿ.

	const. Load	time varying Load
Stat^{n}ary Elements	class 1	class 2
Moving Elements	class 3	class 4

The graph of static loading is as follows:

The graph of dynamic loading is as follows:





II → Failure Models: –

- Funct^{n}al failure
  - → Output is different from the desired value
  - → change in shape, size, & surface finish.
- Material failure
  - → Plastic deformat^n → Breakage of parts.

// → $\boxed{F.O.S = \dfrac{\text{Ultimate Stress}}{\text{Allowable Stress}}}$

// → Theories of Failure :-

① Max. principal stress theory (Rankine Theory)

→ For Brittle Materials

→ $\sigma_1 = \pm\sigma_{oc}$ & $\sigma_2 = \pm\sigma_y$

② Max. principal strain theory (St. Venant's Theory)

→ $E\varepsilon_1 = \sigma_1 - \nu\sigma_2 = \pm\sigma_0$

→ $E\varepsilon_2 = \sigma_2 - \nu\sigma_1 = \pm\sigma_0$

③ Max. shear stress theory (Tresca Theory)

→ For Ductile Materials

→ $\sigma_1 - \sigma_2 = \pm\sigma_y$

→ $\sigma_2 - \sigma_3 = \pm\sigma_y$

→ $\sigma_3 - \sigma_1 = \pm\sigma_y$

④ Max. strain energy theory (Beltrami's Theory)

$$\left(\frac{\sigma_1}{\sigma_y}\right)^2 + \left(\frac{\sigma_2}{\sigma_y}\right)^2 - 2\nu\left(\frac{\sigma_1\sigma_2}{\sigma_y^2}\right) = 1$$

⑤ Distortn energy theory (Von-Mises yield criterion) → For Ductile Materials

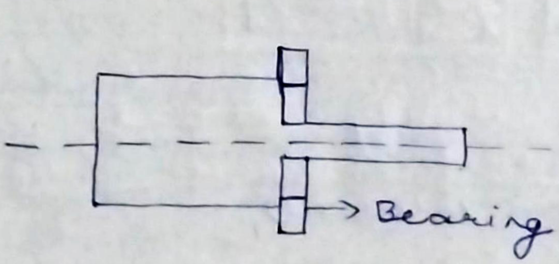
→ $E_T = \frac{1}{2}(\sigma_1\varepsilon_1 + \sigma_2\varepsilon_2 + \sigma_3\varepsilon_3)$

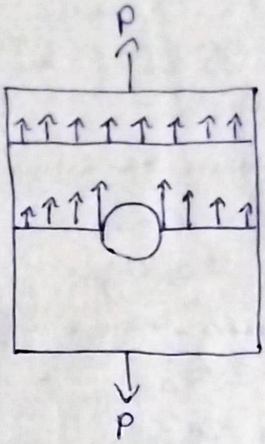
→ $E_V = \frac{3}{2}\sigma_{av}\cdot\varepsilon_{av}$

→ $E_d = E_T - E_V = (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2$
$= 2\sigma_y^2$

N.B It's clear that an immediate assessment of failure can probably be made by plotting any experimental data on the combined yield surface. The failure of ductile materials are most accurately governed by the distortn energy theory, while brittle materials use the max. principal stress theory (IMP)

// → Stress concentratn :-



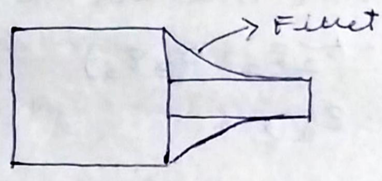


$\therefore$ Stress concentrat^n factor (theoretical) $= K_t = \dfrac{\text{Max. stress}}{\text{Nominal stress}}$

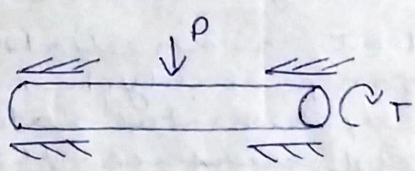
$$\therefore \boxed{\sigma_3 = \sigma_1\left[1 + \frac{2b}{a}\right]}$$

Here, (b) is the width & (a) is the ht. If however (b) is large compared to (a), then the stress at the edge of transverse crack is very large & consequently (K_T) is also large.

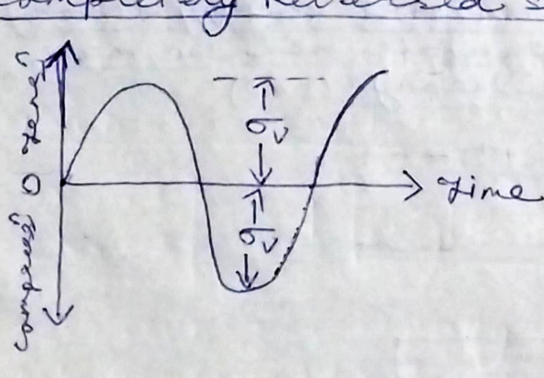
//→ Method of reducing stress concentrat^n:-



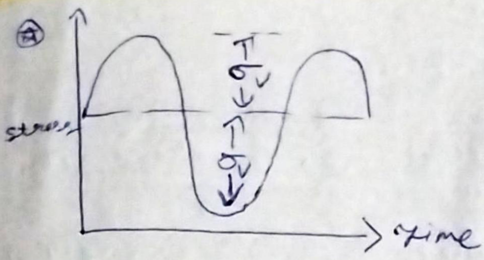
//→ Dynamic loading:-



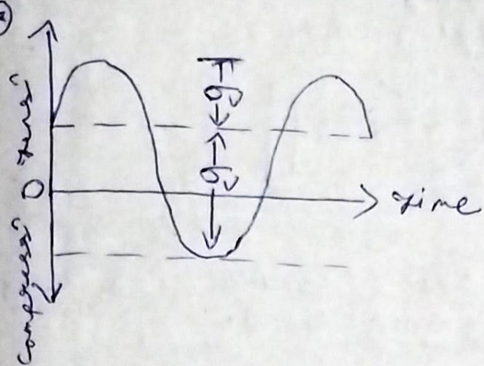
(a) Completely Reversed Stress



Mean value = 0

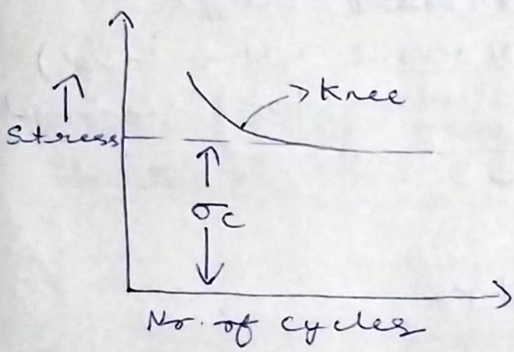


(Repeated stress)



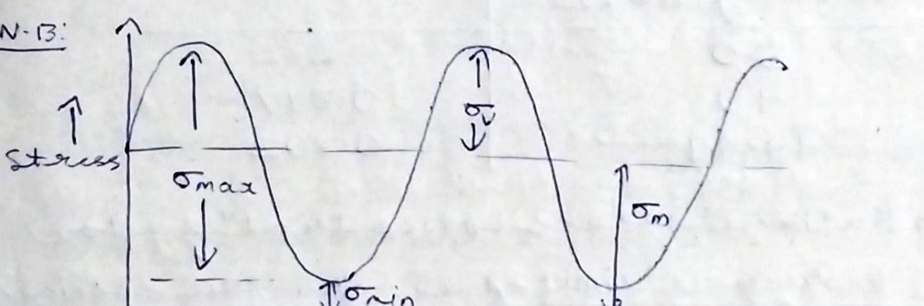
(Fluctuating stress)

// → Endurance or Fatigue Failure :-



(For steel).

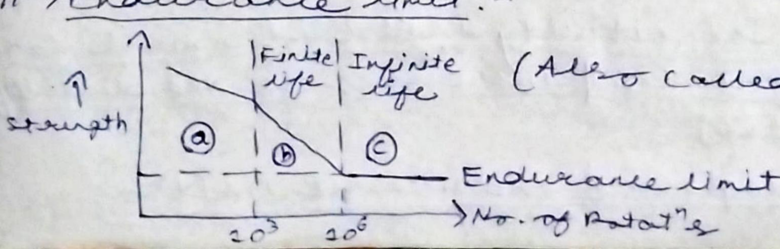
N.B:



$$\boxed{\therefore \sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2}}$$

$$\boxed{\therefore \sigma_{variable} = \frac{\sigma_{max} - \sigma_{min}}{2}}$$

// → Endurance limit :-



(Also called S-N diagram)

Here ⓐ → Low cycle fatigue

ⓑ & ⓒ → High cycle fatigue

N.B: [When 10^6 cycles are considered, then the (Def^n) endurance strength is called endurance limit.]

The above graph is for reversed bending.

The endurance limit needs to be corrected using a no. of factors. In general, the modified endurance limit (σ_e') is given by:

$$\boxed{\sigma_e' = \sigma_e \left[\frac{C_1 C_2 C_3 C_4 C_5}{K_f}\right]}$$

Here:

ⓐ C_1 = Size factor i.e. more is the size, less will be the endurance limit.

ⓑ C_2 = Load factor

$$\therefore C_2 = \begin{cases} 1 & (\text{for reversed bending}) \\ 0.85 & (\text{for reversed axial load}) \\ 0.78 & (\text{for reversed tors}^n\text{al load}) \end{cases}$$

ⓒ C_3 = Surface factor

ⓓ C_4 = Temp. factor

ⓔ C_5 = Reliability factor

∴

Reliability %age	C_5
50	1
90	0.897
99.99	0.702

ⓕ K_f = Fatigue stress concentrat^n factor.

$$= \frac{\text{Endurance limit of a notch-free specimen}}{\text{Endurance limit of a notched specimen}}$$

//→ Notch Sensitivity:-

$$\boxed{q = \frac{K_f - 1}{K_t - 1}} \quad \& \quad \boxed{q \in [0, 0.7]}$$

[The notch sensitivity index is small for ductile materials & increases as the ductility decreases.] (IMP)

//→ How to improve endurance limit:-

* Better surface finish.
* Introduct^n of prestressed surface layer in compress^n.

N.B: Fatigue cracks are due to tensile stress & they propagate under these condit^{ns}. So, format^n of layers stressed in tens^n must be avoided. This can be accomplished by shot blasting, peening, tumbling or cold working by rolling. Carburising, Nitriding & other type of surface coating improve endurance strength.

// → Gerber Curve:-

$$\boxed{\left(\frac{\sigma_v}{\sigma_e}\right) + \left(\frac{\sigma_m}{\sigma_u}\right)^2 = 1}$$

// → Goodman line:-

$$\boxed{\left(\frac{\sigma_v}{\sigma_e}\right) + \left(\frac{\sigma_m}{\sigma_u}\right) = 1}$$

// → Soderberg line:-

$$\boxed{\left(\frac{\sigma_v}{\sigma_e}\right) + \left(\frac{\sigma_m}{\sigma_y}\right) = 1}$$

// → Formulas:-

* Combined loading

(a) $\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

(b) $(\tau_{xy})_{max} =$ largest of $\left|\frac{\sigma_1 - \sigma_2}{2}\right|, \left|\frac{\sigma_2 - \sigma_3}{2}\right|, \left|\frac{\sigma_3 - \sigma_1}{2}\right|$

* Max. shear stress theory
(3D case)

$\frac{\sigma_y}{FOS} =$ largest of $|\sigma_1 - \sigma_2|, |\sigma_2 - \sigma_3|, |\sigma_3 - \sigma_1|$

(a) When σ_1 & σ_2 are opposite in sign, then:

$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$

(b) When σ_1 & σ_2 are of same sign, then:

$\tau_{max} =$ largest of $\left|\frac{\sigma_1}{2}\right|$ or $\left|\frac{\sigma_2}{2}\right|$

* Distort^n Energy theory:-

② 3D Case

$$\left[\frac{\sigma_y}{FOS}\right]^2 = \sigma_1^2 + \sigma_2^2 + \sigma_3^2 - \sigma_1\sigma_2 - \sigma_2\sigma_3 - \sigma_3\sigma_1$$

ⓑ 2D Case

$$\left[\frac{\sigma_y}{FOS}\right]^2 = \sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = \sigma_x^2 + \sigma_y^2 + 3\tau_{xy}^2$$

(i) For brittle material, if σ_1 & σ_2 are of same sign, then the effect of smaller stress is negligible.

$$\therefore \sigma_1 \le \frac{\sigma_{ut}}{FOS} \quad \& \quad \therefore \sigma_1 \le \frac{\sigma_{uc}}{FOS}$$

where, σ_{ut} = Ultimate tensile stress

σ_{uc} = Ultimate compressive stress

(ii) For brittle material, if σ_1 & σ_2 are of opposite sign, then:

$$\boxed{\frac{1}{FOS} = \frac{\sigma_1}{\sigma_{ut}} + \frac{\sigma_2}{\sigma_{uc}}}$$

⊛ Dynamic loading:

ⓐ Soderberg Eqn

$$\frac{1}{FOS} = \frac{\sigma_v}{\sigma_e} + \frac{\sigma_m}{\sigma_y} \quad \text{(For static case)}$$

$$\frac{1}{FOS} = \frac{K_f\sigma_v}{\sigma_e} + \frac{\sigma_m}{\sigma_y} \quad \text{(For dynamic case i.e. tens}^n \text{ \& compress}^n\text{)}$$

ⓑ Goodman Eqn

$$\frac{1}{FOS} = \frac{K_f\sigma_v}{\sigma_e} + \frac{\sigma_m}{\sigma_u} \quad \text{(Ductile materials under dynamic loading)}$$

$$\frac{1}{FOS} = \frac{K_f\sigma_v}{\sigma_e} + \frac{K_t\sigma_m}{\sigma_u} \quad \text{(Brittle materials under dynamic loading)}$$

So, the general eqn. is:

$$\frac{\sigma_y}{FOS} = \sigma_m + K_f\sigma_v\left(\frac{\sigma_y}{\sigma_e}\right) = \sigma_{ne} = \text{Equivalent fatigue stress}$$

⊝ Max shear stress theory:-

$$\tau_e = \frac{\sqrt{(\sigma_{ne})^2 + 4\tau_e^2}}{2}$$

Now, for normal stress theory:

$(\sigma_{ne})_{max} = \frac{1}{2}\sigma_{ne} + \frac{1}{2}\sqrt{\sigma_{ne}^2 + 4\tau_{ne}^2}$

- Distortn energy

$$\frac{\sigma_y}{FOS} = \sqrt{\left(\sigma_m + K_f\sigma_v\frac{\sigma_y}{\sigma_e}\right)^2 + 3\left(\tau_m + K_f\tau_v\frac{\tau_y}{\sigma_e}\right)^2}$$

Q: The stress at a pt. in a body are:
$\sigma_x = 91\ MNm^{-2}$; $\sigma_y = 21\ MNm^{-2}$; $\tau_{xy} = 84\ MNm^{-2}$,
$\sigma_y = 280\ MNm^{-2}$.

Find FOS by max. shear stress theory & distortn energy theory.

Ans: $\sigma_1 = \frac{\sigma_x + \sigma_y}{2} + \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = 147\ MNm^{-2}$

$\sigma_2 = \frac{\sigma_x + \sigma_y}{2} - \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = -35\ MNm^{-2}$

N.B: For ductile material, shear strength at yield pt is 0.5–0.6 times its tensile strength at yield pt.

- $\frac{\sigma_y}{2FOS}$ = largest of $\left|\frac{\sigma_1 - \sigma_2}{2}\right|, \left|\frac{\sigma_1}{2}\right|, \left|\frac{\sigma_2}{2}\right|$

$= 91, 73.5, 17.5\ MNm^{-2}$

So, ($91\ MNm^{-2}$) is taken as the largest value:

Now, $\frac{\sigma_y}{2FOS} = 91 \Rightarrow FOS = 1.54$

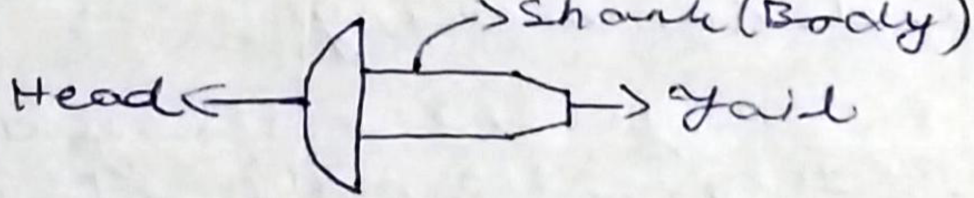
- Again, $\left[\frac{\sigma_y}{FOS}\right]^2 = \sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 \Rightarrow FOS = 1.67$

Mod-2

// -> **Design of Jts** :- It's of 2 types i.e.:

(a) Permanent Jt.   (b) Temporary or Detachable jt.

// -> **Design of riveted jt**:-



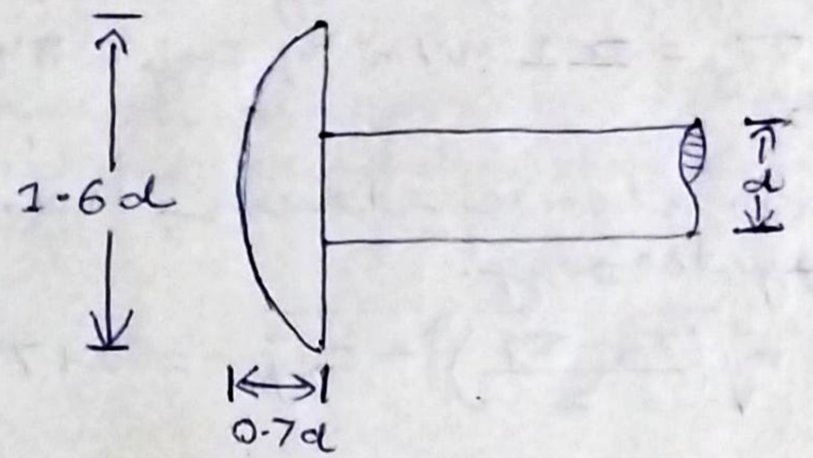
// -> **Methods of Riveting**:-

(IMP) The funct^n of rivets in a jt. is to make a connect^n that has strength & tightness. In structural & pressure vessel riveting, the dia. of rivet hole is usually 1.5mm larger than the nominal dia. of the rivet.

Riveting can be done either at room temp. or at elevated temp. For structural work,

cold riveting is used, & for leakproof joints like pressure vessels, hot riveting is used.

For steel rivets upto 12mm dia, the cold riveting process may be used while for larger dia. rivets, hot riveting process is used.



(Snap head)

II→ types of rivet head:-

⊛ Snap head ⊛ Pan head ⊛ Pan head with tapered neck ⊛ Round counter sunk head 60° ⊛ Flat counter sunk head 60° ⊛ Flat head

→ Snap heads are used mainly for structural work & m/c riveting.

→ Counter-sunk heads are used for ship building where flush surfaces are necess-
-ary.

→ Conical heads are used where riveting is done by hand hammering.

→ Pan heads are req. where very high strength is needed ∵ they've the max. strength, but they're very difficult to shape.

II → Types of Riveted jts:-

- Lap jt.
- Butt jt.

II → Types of Riveted Lap jt. in terms of jt. arrangement:-

- Single riveted lap jt.
- Double riveted lap jt.
- Zig-zag double riveted lap jt. (contd..)

(contd...)

- Triple chain riveted lap jt.
- Triple zig-zag chain riveted lap jt.

// -> Types of butt jt.:-

- Single strap
- Double strap

N.B: Double riveted double strap butt jts may be equal or unequal. If the cover plate width isn't same for top & bottom case, then it's called unequal type. (IMP)

// -> Imp. terms used in riveted jts:-

- Pitch -> It's the dist. from the centre of 1 rivet to the centre of the next rivet measured || to the seam. It's denoted by (p).
- Back Pitch -> It's the ⊥ dist. between the centre lines of the successive rows. It's denoted by (P_b).
- ~~Diagonal Pitch -> It's the dist between the centre lines of the successive rows~~
- Diagonal Pitch -> It's the dist between the centres of rivets in adjacent rows of zig-zag riveted jts. It's denoted by (P_d).
- Margin or Marginal Pitch -> It's the dist. between the centres of rivet hole to the nearest edge of the plate. It's denoted by (m).

// -> Failures of riveted jts:-

- Tearing of plate at an edge (m = 1.5d)
- Tearing of plate across a row of rivets: Due to tensile stress in the main plate, the main or cover plate may tear off across a row of rivets. In such cases, we consider only 1 pitch length of the plate, ∵ every rivet is responsible for that much length of the plate only.

$\therefore \boxed{P_t = \sigma_t (p - d) t}$ → Rivet hole

where, P_t = tensile force.

σ_t = Allowable tensile stress of the plate material.

t = Plate thickness.

Now, $\boxed{\text{Allowable stress} = \dfrac{\text{Material Strength}}{\text{FOS}}}$

Shearing of rivet:

→ ⓐ Single strap → $\boxed{P_s = \tau_s \left(\frac{\pi}{4} d^2\right)}$ → Rivet

→ ⓑ Double strap → $\boxed{P_s = 2\tau_s \left(\frac{\pi}{4} d^2\right)}$

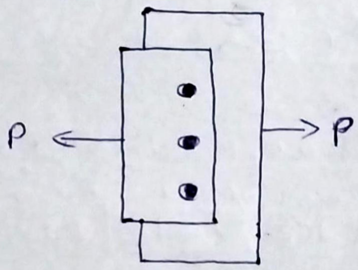
N.B: $\boxed{\tau_s = (0.5 - 0.6)\sigma_t}$ & $\boxed{\sigma_c = (1.2 - 1.4)\sigma_t}$

N.B: As per Indian Boiler Regulatⁿ, Area is given by:

$$\boxed{\text{Area} = 1.875 \frac{\pi}{4} d^2}$$

Now, $\boxed{P_s = n \tau_s \left(\frac{\pi}{4} d^2\right)}$, where (n) = No. of Rivets.

II → **Crushing failure of Rivet:-**



$\therefore \boxed{A_c = dt}$ where,

d → Rivet hole dia.

t → Plate thickness.

σ_c → Safe or Permissible crushing stress for the rivet or plate material.

A_c → Projected Area.

So, the crushing resistance is given by:

$\boxed{P_c = n d t \sigma_c}$ → Rivet hole

// → Efficiency of rivet jt.:-

$$\boxed{\eta = \frac{\text{min. of } P_t, P_s, \& P_c}{\sigma_t p t}}$$

Among (P_t), (P_s), & (P_c), whichever is lowest is the max. force that the jt. can withstand.

	Jts	%age Efficiency
Lap	Single Riveted	50-60
	Double Riveted	60-72
	Triple Riveted	72-80
Butt	Single Riveted	55-60
	Double Riveted	76-84
	Triple Riveted	80-88

// → Design of Riveted Jts:-

- When plate thickness is ≥ 8 mm, use Unwin's formula i.e.

$$\boxed{d = 6\sqrt{t} \text{ mm}}$$

- If plate thickness is < 8 mm, then:

$$\boxed{\sigma_c t = \frac{\pi}{4} d \tau_s} \quad (P_c = P_s)$$

- To find the pitch, use:

$$\boxed{\sigma_t (p-d) t = \tau_s\left(\frac{\pi}{4} d^2\right)} \quad (P_t = P_s)$$

where, $\boxed{p \geq 2.5 d}$

- Margin $(m) = 1.5d$
- Back pitch $(P_b) = 0.33p + 0.67d$
- Diagonal pitch $(P_d) = \sqrt{p^2 + P_b^2}$

Q: 2 plates of 7mm thickness are connected by a double riveted lap jt. of zig-zag pattern. Calculate rivet dia., rivet pitch, & dist. between rows of rivets for the jt. Assume $\sigma_t = 90$ MPa, $\tau_s = 60$ MPa, $\sigma_c = 120$ MPa

Ans - Given, $t = 7$mm < 8mm & $n = 2$

- $P_c = P_s \Rightarrow n d t \sigma_c = n \tau_s\left(\frac{\pi}{4} d^2\right)$

$\Rightarrow \sigma_c t = \frac{\pi}{4} d \tau_s \Rightarrow d = 17.825\,mm \simeq 19\,mm$

$\therefore$ corresponding rivet dia. is 18mm.

(a) $P_t = P_s \Rightarrow \sigma_t (p-d) t = n \tau_s \left(\frac{\pi}{4} d^2\right)$

$\Rightarrow p = 67.47\,mm \simeq 68\,mm$

(a) $P_b = 0.33p + 0.67d = (0.33 \times 68) + (0.67 \times 19)$

$= 35.17\,mm \simeq 36\,mm$

Q: A double riveted lap jt. is made between 15mm thick plates. The rivet dia. & pitch are 25 & 75mm respectively. If the ultimate stress are 400 MPa in tensⁿ, 320 MPa in shear, & 640 MPa in crushing, find the min. force per pitch which will rupture the jt. If the above jt. is subjected to a load such that the F.O.S is 4, find out the actual stresses developed in the plates & the rivets.

Ans- Given, $t = 15\,mm$, $d = 25\,mm$, $p = 75\,mm$; $\sigma_{ut} = 400\,MPa$, $\tau_u = 320\,MPa$, $\sigma_{uc} = 640\,MPa$

$\therefore P_t = (p-d) t \times \sigma_{ut} = (75-25) \times 15 \times 400 = 300\,kN$

$\therefore P_s = n \times \left(\frac{\pi}{4} d^2\right) \tau_s = 2 \times \left(\frac{\pi}{4} \times (25)^2\right) \times 320 = 314.16\,kN$

$\therefore P_c = n d t \sigma_c = 2 \times 25 \times 15 \times 640 = 480\,kN$

Among these 3 forces, the max. force in tearing is 300kN, which is the lowest value. Now, considering the FOS value of 4;

$$\text{Max load in tens}^n = \frac{\text{Max. force in tearing failure}}{\text{FOS}}$$

$$= \frac{300\,kN}{4} = 75\,kN$$

$\therefore P_t = (p-d) t \times \sigma_t \Rightarrow 75000 = (75-25) \times 15 \times \sigma_t$

$\Rightarrow \sigma_t = 100\,N/mm^2$

$\therefore P_s = n\left(\frac{\pi}{4} d^2\right) \tau_s \Rightarrow 75000 = 2\left(\frac{\pi}{4} \times (25)^2\right) \times \tau_s$

$\Rightarrow \tau_s = 76.394\,N/mm^2$

$\therefore P_c = n d t \sigma_c \Rightarrow 75000 = 2 \times 25 \times 15 \times \sigma_c$

$\Rightarrow \sigma_c = 100\,N/mm^2$

$$\therefore \eta = \frac{\text{min. of } P_t, P_s, P_c}{\sigma_t p t} = \frac{300\,kN}{75 \times 15 \times 400} = 0.67 \simeq 67\%$$

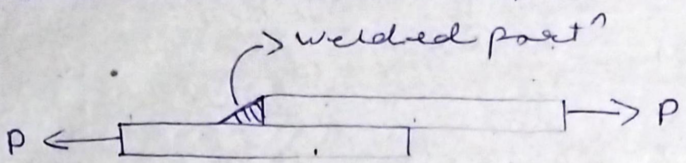
//-> Welded jts:-

Welded jts are of 2 types. They're:

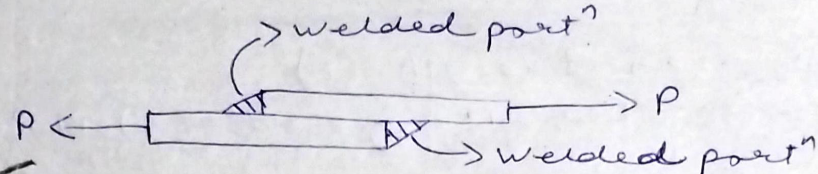
- Lap jt.
- Butt jt.

//-> Types of Lap jt.:-

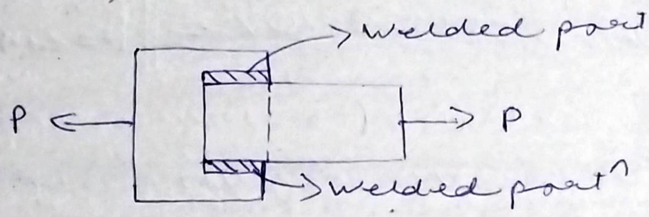
- Single Transverse Lap jt:



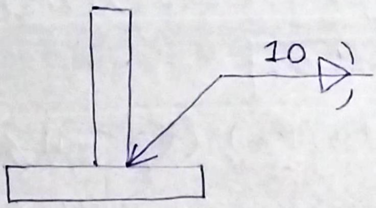
- Double Transverse Lap jt:



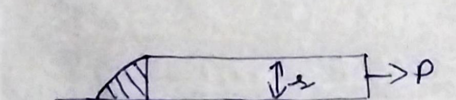
- || fillet Lap jt:

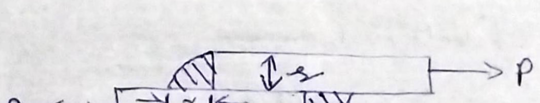


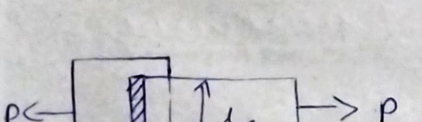
/=> Welding symbol:

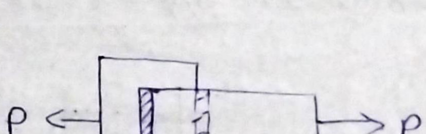


//-> Strength of Transverse fillet welded jts:-



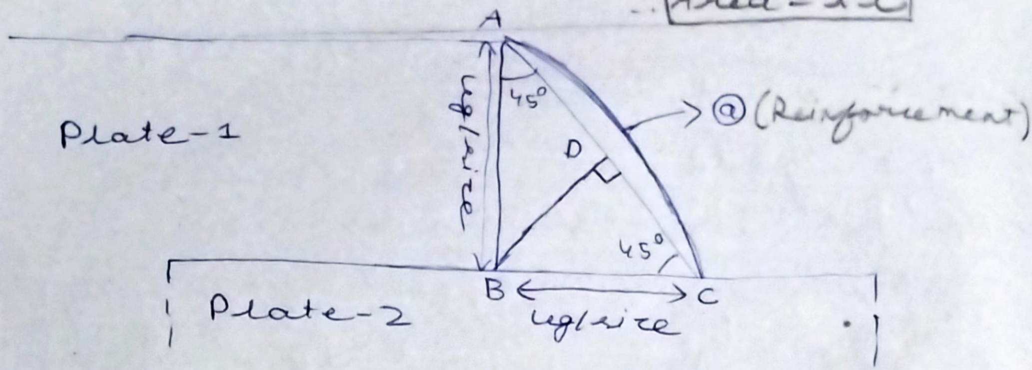






- Single weld
- Double weld

$\therefore \boxed{Area = tl}$



@ [∵ The weld is weaker than the plate due to slag & blowholes ∴ the weld is given a reinforcement which may be taken as 10% of the plate thickness]

Here, BD = throat length = (t)

AB = BC = Leg/size of weld = (s) = Plate thickness

Let, $t = s \sin 45° \Rightarrow t = 0.707\, s$

$\therefore$ $\boxed{\text{Throat thickness } (t) = 0.707\, s}$ [NB: F = sin 45°]

I ⇒ Tensile strength (P_t): (For transverse fillets)

- For single transverse fillet weld

$\boxed{P_t = 0.707\, s\, l\, \sigma_t}$

- For Double transverse fillet weld

$\boxed{P_t = 2 \times 0.707\, s\, l\, \sigma_t}$

I ⇒ Shear strength (P_s): (For ∥ fillets)

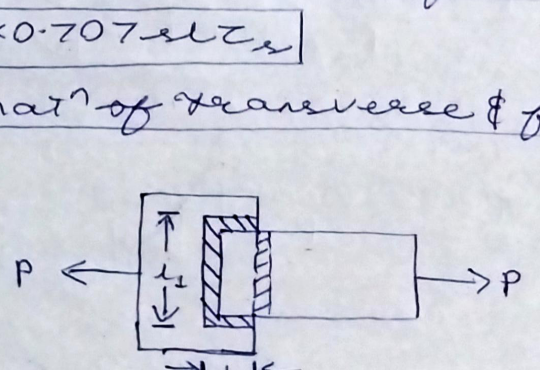
- For single transverse fillet weld

$\boxed{P_s = 0.707\, s\, l\, \tau_s}$

- For Double transverse fillet weld

$\boxed{P_s = 2 \times 0.707\, s\, l\, \tau_s}$

II → Combinatⁿ of transverse & fillet weld:-



$$\therefore \boxed{P = (0.707 s l_2 \sigma_t) + (2 \times 0.707 s l_2 \tau)}$$

- In order to allow for starting & stopping of the bead, 12.5mm should be added to the length of each weld obtained by above expressⁿ.
- For reinforced fillet welds, the throat dimensⁿ may be taken as 0.85t.

Q: A plate 100mm wide & 10mm thick is to be welded to another plate by means of a double ∥ fillets. The plates are subjected to a static load of 80kN. Find the length of weld if the permissible shear stress in the weld doesn't exceed 55 MPa.

Ans- Given, $l_p = 100mm$; s = Weld size = Plate thickness = 10mm ; $P = 80kN$; $\tau_s = 55MPa$

$$\therefore \underline{P = 2 \times 0.707 s l \tau_s}$$

$$\Rightarrow 80 \times 10^3 = 2 \times 0.707 \times 10 \times l \times 55$$

$$\Rightarrow l = 102.867\ mm$$

∴ The weld length would be:

$$l_w = l + 12.5mm = 115.367\ mm\ (Ans)$$

Q: A plate 100mm wide & 12.5mm thick is to be welded to another plate by means of ∥ fillet welds. The plate is subjected to a load of 50kN. Find the weld length so that the max. stress doesn't exceed 56MPa. Consider the jt. 1st under static loading & then under fatigue loading.

Ans- Given, $l_p = 100mm$; s = Weld size = Plate thickness = 12.5mm ; $P = 50kN$; τ_s = 56MPa.

$$\therefore \underline{P = 2 \times 0.707 s l \tau_s}$$

$$\Rightarrow 50 \times 10^3 = 2 \times 0.707 \times 12.5 \times l \times 56$$

$$\Rightarrow l = 50.515\ mm$$

∴ The weld length would be:

$$l_w = l + 12.5mm = 63.015mm$$

Now, the stress concentratⁿ factor for ∥ fillet weld is 2.7.

$$\therefore \text{Permissible shear stress } (\tau_s) = \frac{56}{2.7} = 20.741\ MPa$$

$\therefore P = 2 \times 0.707\, s\, l\, \tau_s$

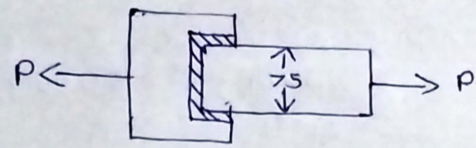
$\Rightarrow 50 \times 10^3 = 2 \times 0.707 \times 12.5 \times l \times 20.741$

$\Rightarrow l = 136.389$ mm

$\therefore$ The weld length would be:

$$l_w = l + 12.5\text{mm} = 148.889 \text{ mm}$$

Q: A plate 75mm wide & 12.5mm thick is joined with another plate by a single transverse weld & a double fillet weld. The max. tensile & shear stresses are 70MPa & 56MPa respectively. Find the length of each ∥ fillet weld if the jt. is subjected to both static & fatigue loading.



Ans- Given, $l_p = 75$mm; s = weld size = Plate thickness = 12.5mm; $\sigma_t = 70$MPa; $\tau_s = 56$MPa

Now, the effective length (l_1) for transverse weld is given by:

$l_1 = 75 - 12.5 = 62.5$ mm

Let, l_2 = length of each ∥ fillet weld

Now, P = Area × Stress $= (75 \times 12.5) \times 70 = 65.625$ kN

(a) Load carried by the single transverse weld is given by:

$P_1 = 0.707\, s\, l_1\, \sigma_t = 0.707 \times 12.5 \times 62.5 \times 70$
$= 38.644$ kN

$\therefore$ Load carried by double ∥ fillet weld is given by:

$P_2 = 2 \times 0.707\, s\, l_2\, \tau_s = 2 \times 0.707 \times 12.5 \times l_2 \times 56$
$= 989.8\, l_2$ N $\simeq 990\, l_2$ N

$\therefore$ Load carried by the jt. is given by:

$P = P_1 + P_2 \Rightarrow l_2 = 27.254$ mm

$\therefore$ The weld length would be:

$$l_w = l_2 + 12.5 = 39.754 \text{ mm} \simeq 40 \text{mm}$$

(b) Now, the stress concentratⁿ factor for transverse weld is 1.5 & for ∥ fillet welds is 2.7.

$\therefore$ Permissible tensile stress $(\sigma_t) = \frac{70}{1.5} = 46.7$ N/mm^2

∴ Permissible shear stress $(\tau_s) = \frac{56}{2.7} = 20.74 \ N/mm^2$

So, $P_1 = 0.707 s l_1 \sigma_t = 0.707 \times 12.5 \times 62.5 \times 46.7$
$= 29.794 \ kN$

$P_2 = 2 \times 0.707 s l_2 \tau_s = 2 \times 0.707 \times 12.5 \times l_2 \times 20.74$
$= 366.58 \, l_2 \ N$

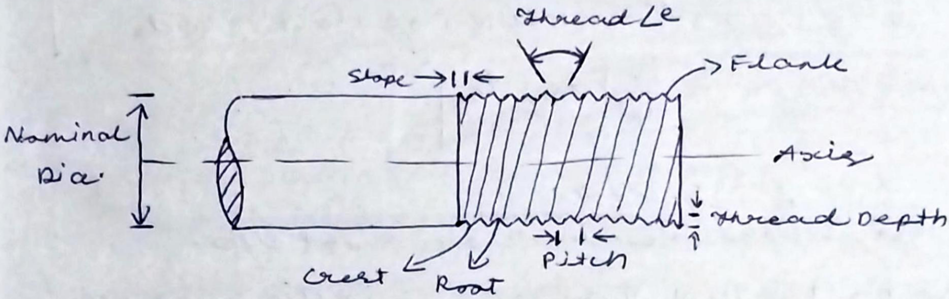
∴ Load carried by the jt. is given by:

$P = P_1 + P_2 \Rightarrow l_2 = 97.744 \ mm$

∴ The weld length would be:

$l_w = l_2 + 12.5 = 110.244 \ mm$

// → Threaded screw jts :-



- Screws are used both to hold things together as fasteners & to move loads as so called power or lead screws. (Def^n)
- Screws as fasteners can be arranged to take tensile loads, shear loads or both.
- 3 std series of thread pitch families are defined in UNS threads:
  - → Coarse pitch (UNC)
  - → Fine threads (UNF)
  - → Extra-Fine pitch (UNEF)

// → Nomenclature :-

- Major dia
- Minor dia
- Pitch dia
- Pitch
- Lead
- Crest
- Root
- Thread depth
- Flank
- Thread ∠e
- Slope

// → Stresses in screwed fastening due to static loading :-

- Internal stresses due to screwing up forces
- Stresses due to external forces
- Stresses due to combinat^n of stresses at ① & ②

$\therefore$ $\boxed{\text{Internal tension } (P_i) = 2840d \text{ N}}$ --- (1)

where, d = Nominal dia of bolt.

Equn. (1) is used for fluid or air-tight jt. For non-fluid or non-air-tight jt, then:

$$\boxed{P_i = 1420d \text{ N}}$$

II → ~~Initial~~ Internal stresses due to screwing-up forces:-

(IMP) Smaller dia. M16 or M18 aren't allowed for fluid or air-tight jts. If the bolt isn't initially stressed, the max. safe axial load which may be applied to it is given by:

$$\boxed{P = \text{Permissible Stress} \times \text{cross-sectional area at the bottom of the thread}}$$

$$\therefore \boxed{\text{Stress Area} = \frac{\pi}{4}\left[\frac{d_p + d_c}{2}\right]^2}$$

where, d_p = pitch dia.

d_c = core or minor or root dia.

N.B: The bolt, stud, & screw usually carry a load in the direction of the bolt axis which induces a tensile stress in the bolt.

$\therefore$ External load $= \boxed{P_t = \frac{\pi}{4} d_c^2 \sigma_t}$, where, $d_c = \sqrt{\frac{4P}{\pi \sigma_t}}$

- If there are n-bolts, then, $\boxed{P_t = \frac{\pi}{4} d_c^2 \sigma_t n}$
- If std table isn't available, then for coarse threads, $\boxed{d_c = 0.84d}$, where (d) is the nominal bolt dia.

II → Shear Stress:-

Bolts are subjected to shearing loads. They must be located in such a way that the shearing load comes upon the bolt body & not upon the threaded portion.

$\therefore$ $\boxed{P_s = \text{Shearing load} = \frac{\pi}{4} d^2 \tau n}$ & $d = \sqrt{\frac{4P_s}{\pi \tau n}}$

II → Combined tension & shear stress:-

- Max. Principal shear stress (τ_{max}):

$$\boxed{\tau_{max} = \frac{\sqrt{\sigma_t^2 + 4\tau^2}}{2}}$$

- Max. Principal tensile stress $(\sigma_t)_{max}$:

$$\boxed{(\sigma_t)_{max} = \frac{\sigma_t}{2} = \frac{\sqrt{(\sigma_t)^2+4\tau^2}}{2}}$$

// → Thread specs:-

M16 × 1.5 means:

- Minor dia = 14.16 mm (Bolt)
  = 14.376 mm (nut).
- Stress area = 167 mm^2
- Pitch = 1.5 mm

// → Cotter Jt. types:-

- Socket & spigot cotter jt.
- Sleeve & cotter jt.
- Gib & cotter jt.

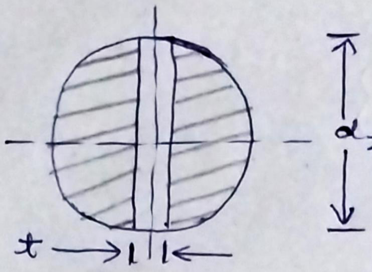
// → Design of socket & spigot cotter jt:-

- P = Load carried by the rods.
- d = Rod dia.
- d_1 = Outside dia. of socket.
- d_2 = Spigot dia. or inside dia. of socket
- d_3 = Outside dia. of spigot collar.
- t_1 = Spigot collar thickness.
- d_4 = Socket collar dia.
- c = Socket collar thickness.
- b = Mean width of cotter.
- t = Cotter thickness.
- l = cotter length.
- a = Dist. from the end of slot to the end of rod.
- σ_t = Permissible tensile stress for rod material.
- τ_s = Permissible shear stress for cotter material.
- σ_c = Permissible crushing stress for the cotter material.

// → Failure of rods in tens^n:-

$$\boxed{P_t = \frac{\pi}{4} d^2 \sigma_t}$$

// → Failure of spigot in tens^n across the weakest sect^n (or slot) :-

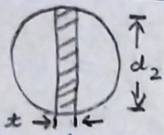


$$\therefore \boxed{P_t = \left[\frac{\pi}{4} d_2^2 - d_2 t\right] \sigma_t}$$

[N·B: D = π/4]

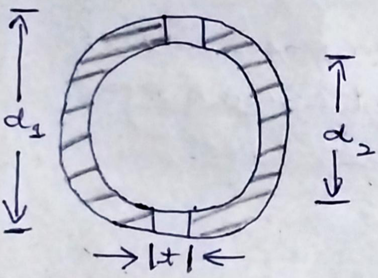
N·B: In actual practice, the cotter thickness is usually taken as $d_2/4$

// → Failure of rod or cotter in crushing :-



$$\boxed{P_c = d_2 t \sigma_c}$$

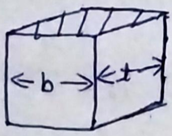
// → Failure of socket in tens^n across the slot :-



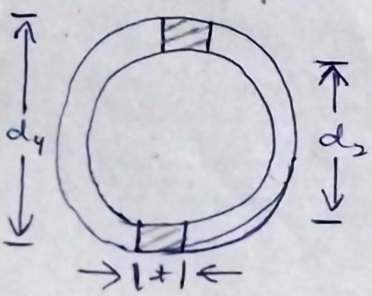
$$\therefore \boxed{P_t = \left[\frac{\pi}{4}(d_1^2 - d_2^2) - (d_1 - d_2)t\right] \sigma_t}$$

// → Failure of cotter in shear :-

$$\therefore \boxed{P_s = 2bt\tau_s}$$



// → Failure of socket collar in crushing :-



$$\therefore \boxed{P_c = (d_4 - d_2) t \sigma_c}$$

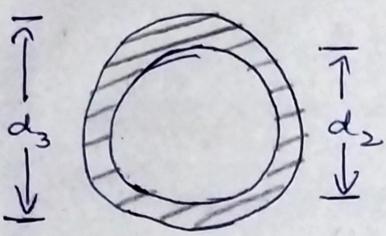
// → Failure of socket end in shearing:-

$$\boxed{P_s = 2(d_4 - d_2)\, c\, \tau_s}$$

// → Failure of rod end (or spigot) in shearing:-

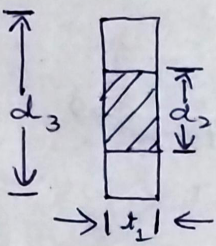
$$\boxed{P_s = 2 a d_2 \tau_s}$$

// → Failure of spigot collar in crushing:-



$$\therefore \boxed{P_c = \frac{\pi}{4}\left[d_3^2 - d_2^2\right] \sigma_c}$$

// → Failure of spigot collar in shearing:-



$$\therefore \boxed{P_s = \pi d_2 t_1 \tau_s}$$

// → Sect^n modulus of cotter:-

$$\boxed{Z = \frac{t b^2}{6}}$$

// → Bending stress induced in cotter:-

$$\sigma_b = \frac{M_{max}}{Z} = \frac{\frac{P}{2}\left[\frac{d_4 - d_2}{6} + \frac{d_2}{4}\right]}{(t b^2)/6} = \boxed{\frac{P(d_4 + 0.5 d_2)}{2 t b^2}}$$

N.B:

- Cotter length is taken as: $\boxed{l = 4d}$
- The taper in cotter must be less than 1 in 24.
- The cotter clearance is generally taken as 2 to 3 mm.
- When all the parts of the jt. are made of steel, the following proport^{n}s in terms of rod dia. (d) are generally adopted:
  (a) $d_1 = 1.75d$ (b) $d_2 = 1.21d$ (c) $d_3 = 1.5d$
  (d) $d_4 = 2.4d$ (e) $a = c = 0.75d$ (f) $b = 1.3d$
  (g) $l = 4d$ (h) $t = 0.31d$ (i) $t_1 = 0.45d$ (j) $e = 1.2d$
- If the rod & cotter are made of steel or

wrought iron, then:

(a) $\tau_s = 0.8\sigma_t$ (b) $\sigma_c = 2\sigma_t$ (IMP)

Q: Design & draw a cotter jt. to support a load varying from 30kN in compression to 30kN in tension. The material used is carbon steel for which the following allowable stresses may be used. The load is applied statically.

(a) Tensile stress = compressive stress = 50MPa

(a) Shear stress = 35MPa

(a) crushing stress = 90MPa

Ans-

(a) Rod dia:

$\therefore P_t = \frac{\pi}{4} d^2 \sigma_t \Rightarrow 30\times10^3 = \frac{\pi}{4} d^2 \times 50$

$\Rightarrow d = 27.63\,mm \approx 28mm$

(a) Spigot dia ~~&~~ & cotter thickness:

$\therefore P_t = \left[\frac{\pi}{4} d_2^2 - d_2 t\right]\sigma_t \Rightarrow 30\times10^3 = \left[\frac{\pi}{4} d_2^2 - \frac{d_2^2}{4}\right] \times 50$

$\Rightarrow d_2 = 33.4mm \approx 34mm$ & $t = \frac{d_2}{4} = 8.5mm$

(a) Induced crushing stress:

$\therefore P_c = d_2 t \sigma_c \Rightarrow 30\times10^3 = 34\times8.5\times\sigma_c$

$\Rightarrow \sigma_c = 103.8\,N/mm^2 > 90MPa$

~~$30\times10^3 = d_2 t \sigma_c$~~

$\therefore P_c = d_2 t \sigma_c \Rightarrow 30\times10^3 = d_2 \times \frac{d_2}{4} \times 90$

$\Rightarrow d_2 = 36.5mm \approx 40mm$ & $t = \frac{d_2}{4} = 10mm$

(a) Outside socket dia:

$\therefore P_t = \left[\frac{\pi}{4}(d_1^2 - d_2^2) - (d_1 - d_2)t\right]\sigma_t$

$\Rightarrow 30\times10^3 = \left[\frac{\pi}{4}(d_1^2 - 40^2) - (d_1 - 40)\times10\right]\times 50$

$\Rightarrow d_1 = 49.4mm \approx 50mm$

(a) Cotter width:

$\therefore P_s = 2bt\tau_s \Rightarrow 30\times10^3 = 2b\times10\times35 \Rightarrow b = 43mm$

(a) Socket collar dia:

$\therefore P_c = (d_4 - d_2)t\sigma_c \Rightarrow 30\times10^3 = (d_4 - 40)\times10\times90$

$\Rightarrow d_4 = 73.3mm \approx 75mm$

(a) Socket collar thickness:

$\therefore P_s = 2(d_4 - d_2)c\tau_s \Rightarrow 30\times10^3 = 2(75-40)\times c\times 35$

$\Rightarrow c = 12mm$

⑦ Dist. from slot end to the rod end:

$\therefore P_s = 2ad_2\tau_s \Rightarrow 30\times10^3 = 2a\times40\times35$

$\Rightarrow a = 10.7mm \approx 11mm$

⑧ spigot collar dia:

$\therefore P_c = \frac{\pi}{4}(d_3^2 - d_2^2)\sigma_c \Rightarrow 30\times10^3 = \frac{\pi}{4}(d_3^2 - 40^2)\times 90$

$\Rightarrow d_3 = 45mm$

⑨ Spigot collar thickness:

$\therefore P_s = \pi d_2 t_1 \tau_s \Rightarrow 30\times10^3 = \pi\times40\times t_1\times35$

$\Rightarrow t_1 = 6.8mm \approx 8mm$

⑩ Cotter length:

$\therefore l = 4d = 4\times28 = 112mm$

⑪ $e = 1.2d = 1.2\times28 = 33.6mm \approx 34mm$

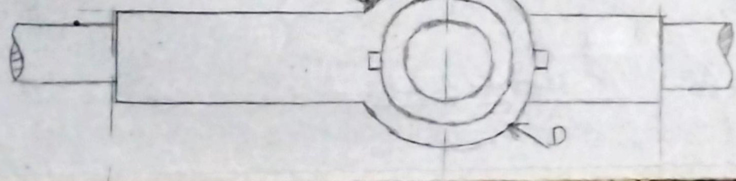
$\therefore$

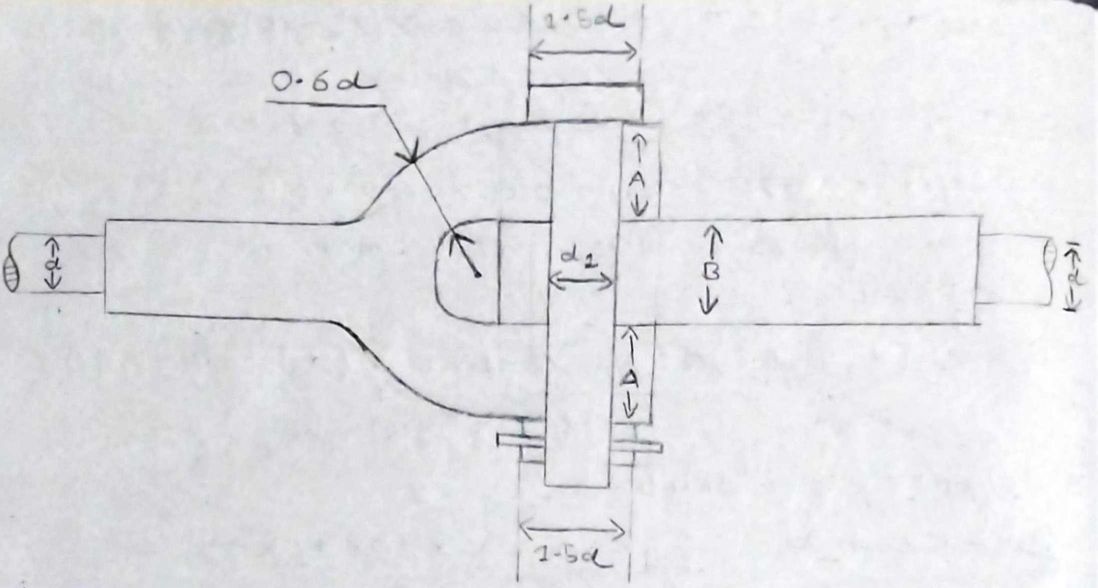
Nomenclature	Value from calc	Value from prop
d_1	50mm	49mm
d_2	40mm	33.88 ≈ 34mm
d_3	45mm	42mm
d_4	75mm	67.2 ≈ 68mm
a	11mm	21mm
c	12mm	21mm
b	43mm	36.4 ≈ 37mm
l	112mm	112mm
t	10mm	8.68 ≈ 9mm
e	34mm	34mm
t_1	8mm	12.6mm ≈ 13mm

// → Knuckle Jt: –

The rods connected by this jt. are subjected to tensile loads, although if the rods are guided, they may support compressive loads as well.

// → Knuckle Jt. proport^{n}s: –





// → Nomenclature :-

- F = Tensile load to be resisted by the jt.
- d = Rod dia. - d_1 = Outside dia. of eye.
- A = thickness of forks. - B = thickness of eye

N.B: B = 2A (for same material)

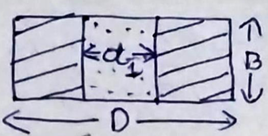
// → Design of knuckle jt. :-

- Tensile failure of rod across the sect^n of dia. (d):

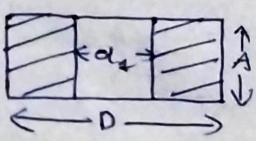
$$\boxed{F = \frac{\pi}{4} d^2 \sigma_t}$$

- Tensile failure of eye:

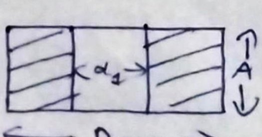
$$\therefore \boxed{F = (D - d_1) B \sigma_t}$$



- Tensile failure of fork:

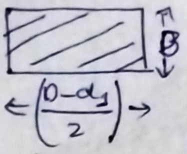


$$\therefore \boxed{F = 2(D - d_1) A \sigma_t}$$

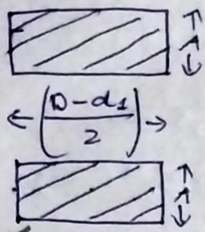


- Shearing failure of eye:

$$\therefore \boxed{F = (D-d_1) B \tau_s}$$



⊛ Shear failure of fork:



$$\therefore \boxed{F = 2(D-d_1) A \tau_s}$$

⊛ Shear failure of pin. It's under double shear

$$\boxed{F = \frac{2\pi}{4} d_1^2 \tau_s}$$

⊛ Crushing between pin & eye:

$$\boxed{F = d_1 B \sigma_c}$$

⊛ Crushing between pin & fork:

$$\boxed{F = 2 d_1 A \sigma_c}$$

Q: Design a knuckle jt. to transmit 150kN. The design stresses may be taken as 75MPa in tens^n, 60MPa in shear, & 150MPa in crushing.

Ans- ⊛ Failure of rod in tens^n:

$\therefore F = \frac{\pi}{4} d^2 \sigma_t \Rightarrow 150 \times 10^3 = \frac{\pi}{4} d^2 \times 75$

$\Rightarrow d = 50.4\text{mm} \approx 52\text{mm}$

⊛ Knuckle pin dia:

$\boxed{d_1 = d} \Rightarrow d_1 = 52\text{mm}$

⊛ Outer dia. of eye:

$\boxed{d_2 = 2d} \Rightarrow d_2 = 104\text{ mm}$

⊛ Dia of knuckle pin head & collar:

$\boxed{d_3 = 1.5d} \Rightarrow d_3 = 78\text{ mm}$

⊛ Thickness of single eye or rod end:

$\boxed{t = 1.25d} \Rightarrow t = 65\text{mm}$

- **Fork thickness:**

$\boxed{t_1 = 0.75d} \Rightarrow t_1 = 39\,mm \approx 40\,mm$

- **Pin head thickness or collar thickness:**

$\boxed{t_2 = 0.5d} \Rightarrow t_2 = 26\,mm$

Now,

- **Failure of knuckle pin in shear:**

$\therefore F = 2\frac{\pi}{4}d_1^2 \tau_s \Rightarrow 150\times10^3 = \frac{2\pi}{4}\times(52)^2\times\tau_s$

$\Rightarrow \tau_s = 35.3\,MPa < 60\,MPa$

- **Failure of single eye or rod end in tension:**

$\therefore F = (d_2 - d_1)t\sigma_t \Rightarrow 150\times10^3 = (104-52)\times65\times\sigma_t$

$\Rightarrow \sigma_t = 44.4\,MPa < 75\,MPa$

- **Failure of single eye or rod in shearing:**

$\therefore F = (d_2 - d_1)t\tau_s \Rightarrow 150\times10^3 = (104-52)\times65\times\tau_s$

$\Rightarrow \tau_s = 44.4\,MPa < 60\,MPa$

- **Failure of single eye or rod in crushing:**

$\therefore F = d_1 t \sigma_c \Rightarrow 150\times10^3 = 52\times65\times\sigma_c$

$\Rightarrow \sigma_c = 44.4\,MPa < 150\,MPa$

- **Failure of forked end in tension:**

$\therefore F = (d_2 - d_1)2t_1\sigma_t \Rightarrow 150\times10^3 = (104-52)\times2\times40\times\sigma_t$

$\Rightarrow \sigma_t = 36\,MPa < 75\,MPa$

- **Failure of forked end in shear:**

$\therefore F = (d_2 - d_1)2t_1\tau_s \Rightarrow 150\times10^3 = (104-52)\times2\times40\times\tau_s$

$\Rightarrow \tau_s = 36\,MPa < 60\,MPa$

- **Failure of forked end in crushing:**

$\therefore F = d_1\times2t\times\sigma_c \Rightarrow 150\times10^3 = 52\times2\times40\times\sigma_c$

$\Rightarrow \sigma_c = 36\,MPa < 150\,MPa$

From above, the induced stresses are less than the given design stresses, $\therefore$ the jt. is safe.

11 → **Shaft Coupling:-**

There're 2 types of shaft coupling, including:

ⓐ **Rigid coupling**

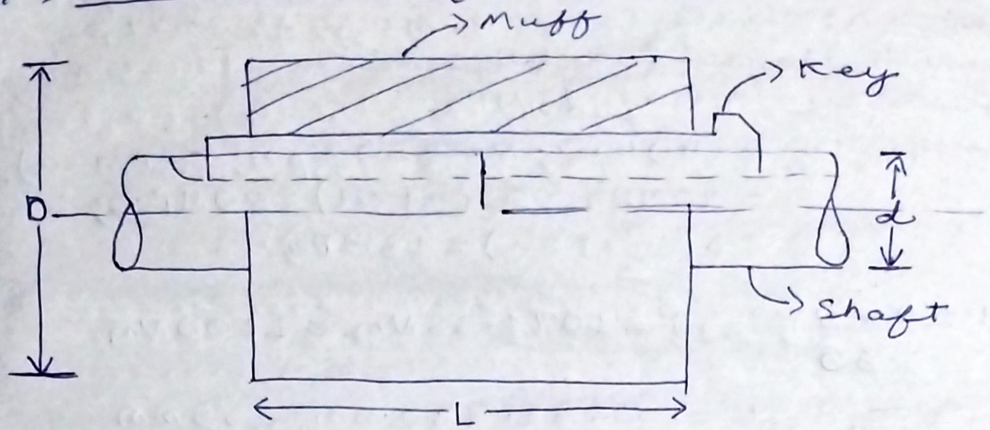
→ Sleeve or muff coupling.

→ Clamp or split muff or compression coupling.

→ Flange coupling.

- Flexible coupling
  - → Bushed pin type coupling.
  - → Universal coupling.
  - → Oldham coupling.

1) → Sleeve or muff coupling :-



- Outer dia of sleeve $(D) = 2d + 13\,mm$
- Sleeve length $(L) = 3.5d$

Let, T = Torque to be transmitted by the coupling.

τ_c = Permissible shear stress of sleeve material i.e. Cast Fe.

N.B: If the safe value of shear stress for Cast Fe isn't given, then it may be taken as 14 MPa (IMP)

- $T = \frac{\pi}{16} \tau_s d^3$ (For shaft) [N.B: $C = \frac{\tau}{\sigma_s}$]
- $T = \frac{\pi}{16} \tau_c \left[\frac{D^4 - d^4}{D} \right]$ (For Muff)
- If 1 key is used, then the key length (l) may be taken as (L).
- If 2 keys are used, then the key length (l) may be taken as $(L/2)$.
- $T = \frac{l w \tau_s d}{2}$ (For shearing of key)
- $T = \frac{l t \sigma_c d}{4}$ (For crushing of key)
- $P = \frac{2\pi N T}{60}$ (Relatⁿ between Power & Torque)

[N.B: $\omega = \frac{2\pi}{60}$]

Q: Design & make a neat dimension'ed sketch of a muff coupling which is used to connect 2 steel shafts transmitting 40kW at 350 rpm. The material for the shafts & key is plain carbon steel for which allowable shear & crushing stresses may be taken as 40MPa & 80MPa respectively. The material for the muff is Cast Fe, for which, the allowable shear stress may be assumed as 15 MPa.

Ans- Given, $P = 40kW$; $N = 350$ rpm; τ_s (steel) $= 40MPa$; σ_c (steel) $= 80MPa$; τ_s (Cast Fe) $= 15MPa$

- $P = \frac{2\pi NT}{60} \Rightarrow T = 1091.35\,Nm \approx 1100\,Nm$

- $T = \frac{\pi}{16}\tau_s d^3 \Rightarrow d = 51.932\,mm \approx 55\,mm$

- $D = 2d + 13mm = 123mm \approx 125mm$

- $L = 3.5d = 192.5mm \approx 195mm$

- $T = \frac{\pi}{16}\tau_s\left[\frac{D^4 - d^4}{D}\right] \Rightarrow \tau_s = 2.98\,N/mm^2 < 15MPa$

∴ Induced shear stress is less than the permissible limit. So, the muff design is safe.

- From databook, for 55mm shaft dia, the key width (w) = 18mm. ⇒ (t) = 18mm

- $l = \frac{L}{2} = 97.5\,mm$

- $T = \frac{lw\tau_s d}{2} \Rightarrow \tau_s = 22.792\,N/mm^2 < 40MPa$

∴ Induced shear stress is less than the permissible limit. So, the key design is safe.

- $T = \frac{lt\sigma_c d}{4} \Rightarrow \sigma_c = 45.584\,N/mm^2 < 80MPa$

∴ Induced ~~shear~~ crushing stress is less than the permissible limit. So, the key design is safe.

II → Flange Coupling:-

There're different types of flange coupling, including:

- Unprotected type flange coupling.
- Protected type flange coupling.
- Marine type flange coupling.

I ⇒ Unprotected flange coupling :-

The usual proport^ns for an unprotected type Cast Fe flange couplings are as follows:

- d = Shaft dia = Inner dia. of hub.
- D = Outer dia. of hub. = $2d$.
- D_1 = Pitch circle dia. of bolts = $3d$.
- D_2 = Outside dia of flange = $4d$.
- t_f = Flange thickness (t_f) = $0.5d$.
- n = No. of Bolts = $\begin{cases} 3, & d \leq 40\text{mm} \\ 4, & d \leq 100\text{mm} \\ 6, & d \leq 180\text{mm} \end{cases}$

I ⇒ Protected flange coupling :-

- t_p = Protected flange thickness = $0.25d$.

I ⇒ Solid forged or Marine type flange coupling :-

- t_f = Flange thickness = $d/3$
- Bolt taper = 1 in 20 to 1 in 40
- Pitch circle dia. of bolts (D_1) = $1.6d$
- D_2 = Outside dia. of flange = $2.2d$

II → Design of flange coupling :-

- n = No. of bolts.
- τ_s, τ_b, τ_k = Allowable shear stress for shaft, bolt, & key material respectively.
- τ_c = Allowable shear stress for the flange material i.e. Cast Fe.
- σ_{cb} & σ_{ck} = Allowable crushing stress for bolt & key material respectively.
- $T = \frac{\pi}{16} \tau_s d^3$ (For shaft)
- $L = 1.5d$ (Hub length)
- The key is designed with usual proport^ns & then checked for shearing & crushing stress. The material of key is usually the same as that of shaft. The length of key is taken equal to the length of hub.

- $T = \frac{\pi D^2}{2} \tau_c t_f$ (For flange)

where, τ_c = Shear stress in flange

t_f = Flange thickness

- Load on all bolts (P_s) = $\frac{\pi}{4} d_1^2 \tau_b n$

- $T = \frac{\pi}{4} d_1^2 \tau_b n \times \frac{D_1}{2}$ (For Bolts)

- Crushing strength of all bolts (P_c) = $n d_1 t_f \sigma_{cb}$

where, d_1 = Nominal bolt dia.

- $T = n d_1 t_f \sigma_{cb} \times \frac{D_1}{2}$ (For Bolts)

Q: Design a cast Fe protective type flange coupling to transmit 15kW of power at 900-RPM from an electric motor to a compressor. The service factor may be assumed as 1.35. The following permissible stresses may be used

- Shear stress for shaft, bolt & key material = 40MPa.
- Crushing stress for bolt & key = 80MPa.
- Shear stress for cast Fe = 8MPa

Ans- Given, $P = 15kW$; $N = 900 rpm$; ~~FOS = 1.35~~;
$SF = 1.35$; $\tau_s = \tau_b = \tau_k = 40MPa$;
$\sigma_{cb} = \sigma_{ck} = 80MPa$; $\tau_c = 8MPa$

- $P = \frac{2\pi NT}{60} \Rightarrow T = 159.155\ Nm$

- $T_{max} = S.F \times T = 214.859\ Nm \approx 215 \times 10^3\ Nmm$

- $T_{max} = \frac{\pi}{16} \tau_s d^3 \Rightarrow d = 30.138mm \approx 35mm$

- $D = 2d = 70mm$

- $L = 1.5d = 52.5mm$

- $T_{max} = \frac{\pi}{16} \tau_c \left[\frac{D^4 - d^4}{D}\right] \Rightarrow \tau_c = 3.405\ MPa < 8MPa$

∴ Induced shear stress is less than the permissible limit. So, the hub design is safe

- From databook, for 35mm shaft dia, the key width (w) = 12mm ⇒ (t) = 12mm
[N.B: $\sigma_{ck} = 2\tau_k$]

- $l = L = 52.5mm$

- $T_{max} = \frac{l w \tau_k d}{2} \Rightarrow \tau_k = 19.5MPa < 40MPa$

∴ Induced shear stress is less than the permissible limit. So, the key design is safe.

- $T_{max} = \frac{l t \sigma_{ck} d}{4} \Rightarrow \sigma_{ck} = 39\,MPa < 80\,MPa$

∴ Induced crushing stress is less than the permissible limit. So, the key design is safe.

- $t_f = 0.5d = 17.5\,mm$
- $T_{max} = \frac{\pi D^2}{2} \tau_c t_f \Rightarrow \tau_c = 1.596\,MPa < 8\,MPa$

∴ Induced shear stress is less than the permissible limit. So, the flange design is safe.

- Let, d_1 = Nominal bolt dia.
  $n = 3$ for 35mm shaft dia.
- $D_1 = 3d = 105\,mm$
- $T_{max} = \frac{\pi}{4} d_1^2 \tau_b n \times \frac{D_1}{2} \Rightarrow d_1 = 6.592\,mm \approx M8$
- $T_{max} = n d_1 t_f \sigma_{cb} \times \frac{D_1}{2} \Rightarrow \sigma_{cb} = 11.838\,MPa < 80\,MPa$

∴ Induced crushing stress is less than the permissible limit. So, the bolt design is safe.

- $D_2 = 4d = 140\,mm$
- $t_p = \cancel{0}.25d = 8.75\,mm \approx 10\,mm$

11→ Springs :-

(Def^n) It's an elastic body, whose funct^n is to distort when loaded & recover its original shape when the load is removed.

⇒ Types of spring :-

- Helical spring.
- Conical & volute spring.
- Tors^n spring (Helical & Spiral)
- Laminated/leaf/flat/carriage spring.
- Disc or ~~Belicalle~~ Belleville spring (these springs consist of a no. of conical discs held together against slipping by a central bolt or tube).

⇒ Terms used in compress^n spring :-

- Solid length → when the compress^n spring is compressed until the coils come in contact with each other, then the spring

is said to be solid.

$$\therefore \boxed{L_s = nd}$$

where, n = No. of coils.
d = Spring wire dia.

- Free length → It's the length of the spring in the free or unloaded conditⁿ. It's given by:

L_F = Solid length + Max compressⁿ + clearance between adjacent coils (or clash allowance)

$= nd + \delta_{max} + 0.15\,\delta_{max}$

$$\Rightarrow \boxed{L_F = nd + (1.15)\,\delta_{max}}$$

- Spring index → It's the ratio of mean dia. of coil to the spring wire dia.

$$\therefore \boxed{C = \frac{D}{d}}$$

where, D = Mean dia. of spring coil.

- Spring rate or spring const. or spring stiffness → It's the load req. per unit deflectⁿ of the spring.

$$\therefore \boxed{k = \frac{W}{\delta}}$$

where, W = Applied load
δ = Spring deflectⁿ.

- Pitch → It's the axial dist. between adjacent coils in uncompressed state.

$$\therefore \boxed{p = \frac{\text{Free length}}{n-1}}$$

1 ⇒ Design of Spring:-

- Under τ^{um}, $T = \dfrac{WD}{2} = \dfrac{\pi}{16}\tau_1 d^3$

$$\Rightarrow \boxed{\tau_1 = \frac{8WD}{\pi d^3}}$$

- τ_2 = Direct shear stress $= \dfrac{4W}{\pi d^2}$.

- τ = Resultant shear stress $= \tau_1 \pm \tau_2$

- Max. shear stress induced in the wire =

$$\frac{8WD}{\pi d^3}\left[1+\frac{d}{2D}\right] = \frac{8WD}{\pi d^3}\left[1+\frac{1}{2C}\right] = \boxed{K_s \times \frac{8WD}{\pi d^3}} = \tau$$

where, $\boxed{K_s = \text{shear stress factor} = 1 + \frac{1}{2C}}$

This formula is used when the spring curvature isn't considered.

- Max. shear stress induced in the wire =

$$\boxed{\tau = K_w \times \frac{8WD}{\pi d^3}} \quad \text{where, } \boxed{K_w = \frac{4C-1}{4C-4} + \frac{0.615}{C}}$$

Here, K_w = Wahl's stress factor.

This formula is used when the spring curvature is considered.

// → **Deflectⁿ of Helical Spring :-**

- Total active wire length $(l) = \pi D n$
- Let, θ = ∠ular deflectⁿ of the wire when acted upon by the torque (T).

∴ $\boxed{\text{Axial deflect}^n\ (\delta) = \frac{\theta D}{2}}$

- We know that:

$$\frac{T}{J} = \frac{\tau}{D/2} = \frac{G\theta}{l}$$

Now, $J = \frac{\pi}{34} d^4$ & G = Rigidity modulus of spring wire material.

- ∴ $\boxed{\theta = \frac{16WD^2n}{Gd^4}}$ & $\boxed{\delta = \frac{8WD^3n}{Gd^4}}$

∴ $\frac{W}{\delta}$ = Spring stiffness = $\frac{Gd^4}{8D^3n}$ = Const

// → **Buckling of spring :-**

$$\boxed{W_{cr} = k \times K_B \times L_F}$$

where, k = spring stiffness = $\frac{W}{\delta}$

L_F = Free length of spring

$\boxed{K_B = \text{Buckling factor} = \frac{L_F}{D}}$ <3

// → **Surge in Spring :-**

$$\boxed{f_n = \frac{d}{2\pi D^2 n}\sqrt{\frac{6Gg}{\rho}} \quad \text{Hertz}}$$

where, d = Spring wire dia
D = Mean dia of spring
n = No. of turns
G = Rigidity modulus
g = Accelerat^n due to gravity
ρ = Density of spring material

Q: A helical spring is made from a wire of 6mm dia & has an outside dia of 75mm. If the permissible shear stress is 350MPa & modulus of rigidity is 84kN/mm², find the axial load which the spring can carry & deflect^n per active turn.

Ans- Given, $d = 8mm$; $D_0 = 75mm$; $\tau = 350\ MPa$;
$G = 84\ kN/mm^2$;

- Mean dia of spring $(D) = D_0 - d = 75 - 6 = 69mm$
- Spring index $(C) = \frac{D}{d} = 11.5$
- Neglecting the effect of curvature, we've:

$$K_s = 1 + \frac{1}{2C} = 1.043$$

- Max. shear stress $(\tau) = K_s \times \frac{8WD}{\pi d^3}$

$\Rightarrow 350 = K_s \times \frac{8WD}{\pi d^3} \Rightarrow W = 412.7N$

- Spring deflect^n $(\delta) = \frac{8WD^3 n}{Gd^4} \Rightarrow \frac{\delta}{n} = \frac{8WD^3}{Gd^4}$

$\Rightarrow \frac{\delta}{n} = 9.96mm$

- Considering the effect of curvature, we've:

$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = 1.12$$

- $\tau = K \times \frac{8WC}{\pi d^2} \Rightarrow W = 383.4N$

- $\delta = \frac{8WD^3 n}{Gd^4} \Rightarrow \frac{\delta}{n} = \frac{8WD^3}{Gd^4} = 9.26mm$

Q: Design a spring for a balance to measure 0-1kN over a scale of length 80mm. The spring is ~~to be~~ to be ~~employed~~ enclosed in a casing of 25mm dia. The approx. no. of turns is 30. The modulus of rigidity is 85kN/mm². Also, calculate the max. shear stress induced.

Ans- Here, $D_0 = D + d$ must be less than 25mm

- $\delta = \frac{8WC^3 n}{Gd} \Rightarrow \frac{C^3}{d} = 28.3$

- Let's assume $d = 4mm$

$\therefore c^3 = 28.3d \Rightarrow c = 4.84$

$\therefore c = \frac{D}{d} \Rightarrow D = 19.36mm$

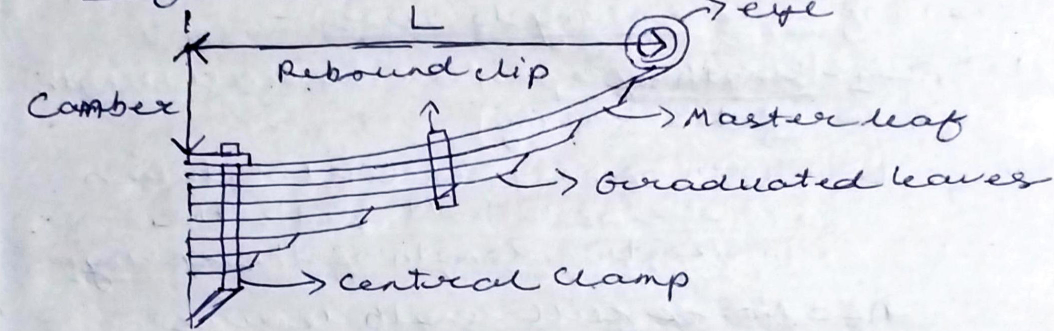
$\therefore D_o = D + d = 23.36 < 25mm$

- Considering the curvature effect, we've:

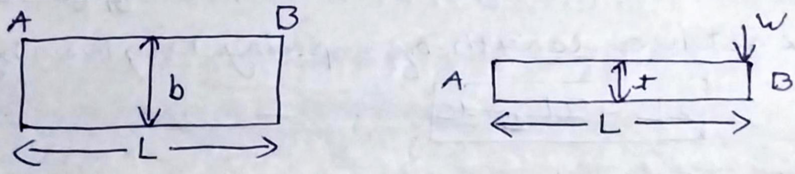
$$K = \frac{4C-1}{4C-4} + \frac{0.615}{C} = 1.322$$

- $\tau = K \times \frac{8WC}{\pi d^2} = 1018.2 MPa$

II → <u>Leaf spring</u>:-



[The leaf spring may carry lateral loads, brake torque, driving torque, etc in additⁿ to the shocks.]



$\therefore$ Let, t = plate thickness

b = plate width

L = plate length or dist. of load (W) from the cantilever end.

- Bending Moment (M) = WL
- Sectⁿ Modulus (Z) = $\frac{I}{y}$

$\therefore Z = \frac{bt^3}{12} \div \frac{t}{2} \Rightarrow \boxed{Z = \frac{bt^2}{6}}$

- Bending Stress (σ) = $\frac{M}{Z}$

$\therefore \sigma = WL \div \frac{bt^2}{6} \Rightarrow \boxed{\sigma = \frac{6WL}{nbt^2}}$

- Deflectⁿ (δ) = $\frac{WL^3}{3EI}$ or $\frac{4WL^3}{nEbt^3}$

The above 4 formulas are for cantilever beam.

- Bending Moment $(M) = WL$
- Sectⁿ Modulus $(Z) = \frac{bt^2}{6}$
- Bending stress $(\sigma) = \frac{M}{Z} = \frac{6WL}{nbt^2}$
- Deflectⁿ $(\delta) = \frac{WL^3}{3EI}$ or $\frac{4WL^3}{nEbt^3}$

The above 4 formulas are for simply supported beam.

1) Length of leaf spring leaves :-

Let, $2L_1$ = Length of span or overall length of the spring.

l = Width of band or dist. between centres of U-bolts. It's the in effective length of the spring.

n_F = No. of full length leaves.

n_G = No. of graduated leaves.

n = Total no. of leaves $= n_F + n_G$

∴ The effective length of spring is given by:

$$\boxed{2L = 2L_1 - l}$$

Q: A truck spring has 12 no. of leaves, 2 of which are full length leaves. The spring supports are 1.05m apart & the central band is 85mm wide. The central load is to be 5.4kN with a permissible stress of 280 MPa. Determine the thickness & width of the steel spring leaves. The ratio of the total depth to the width of the spring is 3. Also determine the deflectⁿ of the spring.

Ans- Let, t = Leaf thickness

b = Leaf width

- $\frac{nt}{b} = 3 \Rightarrow b = \frac{nt}{3} = \frac{12t}{3} = 4t$
- Effective length $(2L) = 2L_1 - l$

$= 1050 - 85 = 965\text{mm}$

$\Rightarrow L = \frac{965}{2} = 482.5\,mm$

* No. of graduated leaves $(n_G) = n - n_F$
$= 12 - 2 = 10$

* Assuming that the leaves are initially stressed, we've:

$$\sigma_F = \frac{18WL}{bt^2(2n_G+3n_F)} = \frac{18 \times \frac{5.4\times10^3}{2} \times 482.5}{4t \times t^2 \times (2\times10 + 3\times2)}$$

$$\Rightarrow 280 = \frac{225476}{t^3} \Rightarrow t = 9.3\,mm \approx 10\,mm$$

$\therefore b = 4t \Rightarrow b = 40\,mm$

* Spring deflectⁿ $(\delta) = \frac{12WL^3}{Ebt^3(2n_G+3n_F)}$

$$= \frac{12 \times \frac{5.4\times10^3}{2} \times (482.5)^3}{2.1\times10^5 \times 40 \times 10^3 \times (2\times10+3\times2)} = 16.7\,mm$$

11 → <u>Keys</u>:-

The functⁿ of key is to transmit power from shaft to hub & to protect the shaft & hub from sudden load.

1 ⇒ <u>Types of keys</u>:-

* Sunk keys * Saddle keys * Tangent keys
* Round keys * Splines

1 ⇒ <u>Sunk keys</u>:-

These are always rectangular or square in cross sectⁿ.

* Width $(w) = \frac{d}{4}$ * Thickness $(t) = \frac{d}{6}$

where, d = shaft dia.

Such keys have taper in 1 in 100 on the top side only.

Sunk keys are of 2 types:
* ∥ sunk key * Gib head key

(IMP)

N.B: * Saddle keys are of 2 types i.e. flat & hollow saddle key (IMP)

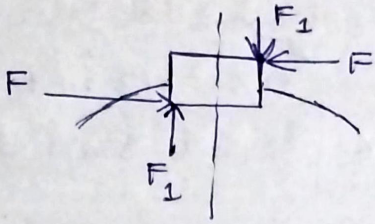
* Round keys are circular in cross sectⁿ.

* Splines are made integral with the shaft. These usually have 4, 6, 10, 16

splines

/=> Forces acting on a ~~spline~~ sunk key:-

- F_1 = compressive force.
- F = shear & crushing force.



/=> Strength of a sunk key:-

Let, T = Torque transmitted by the shaft

F_t = Tangential force acting at the circumference of the shaft.

d = Shaft dia.

l = Key length.

w = Key width.

t = Key thickness.

τ & σ_c = shear & crushing stresses of the key material.

- $F_t = lw\tau$ & $\boxed{T = \frac{F_t d}{2} = \frac{lw\tau d}{2}}$
- F_c = crushing force $= l \times \frac{t}{2} \times \sigma_c = \frac{lt\sigma_c}{2}$
- Torque transmitted $(T) = \frac{F_c d}{2} = \frac{lt\sigma_c d}{4}$
- For square key, shearing & crushing resistance is equal. (IMP)

$\therefore \frac{lw\tau d}{2} = \frac{lt\sigma_c d}{4} \Rightarrow \boxed{\frac{w}{t} = \frac{\sigma_c}{2\tau}}$

N.B: The permissible crushing stress for the usual key material is atleast twice the permissible shearing stress. ($\sigma_c \geq 2\tau$)

- The shearing strength of key is always equal to the torsional shear strength of the shaft. (IMP)

$\therefore$ For key: $T = \frac{lw\tau d}{2}$

$\therefore$ For shaft: $T = \frac{\pi}{16}\tau_1 d^3$

$$\therefore \frac{l w \tau d}{2} = \frac{\pi}{16}\tau_1 d^3 \Rightarrow \boxed{l = 1.571 d \times \frac{\tau_s}{\tau_k}}$$

=> Shafts:-

- Shaft carries twisting & bending moment simultaneously.
- Axle carries only bending moment. It may be stat^{n}ary.
- Spindle is a short shaft that imparts mot^n either to a cutting tool or to a workpiece.

=> Types of shaft:-

- Transmiss^n Shafts -> These shafts transmit power between the source & the m/c's absorbing power.
- M/c shafts -> These form an integral part of m/c itself e.g. crankshaft.

=> Stresses in shaft:-

The following stresses are induced in shafts:

- Shear stresses due to the transmiss^n of torque (i.e. due to tors^{n}al load).
- Bending stresses (tensile or compressive) due to the forces acting upon the m/c elements like gears, pulleys, etc. as well as due to the wt. of shaft itself.
- Stresses due to combined tors^{n}al & bending loads.

N.B: Shafts may be designed on the basis of strength, rigidity, & stiffness.

=> Shafts subjected to twisting moment only:-

We know that: $\boxed{\frac{T}{J} = \frac{\tau}{r}}$

- **For solid shaft:**

$$J = \frac{\pi}{32} d^4 \quad \therefore T = \frac{\pi d^3}{16}\tau \Rightarrow \boxed{T = \frac{\pi}{16}\tau d^3}$$

- **For hollow shaft:**

$$J = \frac{\pi}{32}\left[d_o^4 - d_i^4\right] \quad \therefore \boxed{T = \frac{\pi}{16}\tau\left[\frac{d_o^4 - d_i^4}{d_o}\right]}$$

$$\Rightarrow T = \frac{\pi}{16}\tau\frac{d_o^3}{d_o^3}\left[\frac{d_o^4 - d_i^4}{d_o}\right]$$

$$= \frac{\pi}{16}\tau(d_o)^3\left[\frac{d_o^4 - d_i^4}{d_o^4}\right]$$

$$= \frac{\pi}{16}\tau(d_o)^3\left[1 - \left(\frac{d_i}{d_o}\right)^4\right]$$

$$\Rightarrow \boxed{T = \frac{\pi}{16}\tau(d_o)^3\left[1 - k^4\right]}, \text{ where, } \boxed{k = \frac{d_i}{d_o}}$$

$\Rightarrow$ Solid & Hollow shafts transmitting same power & torque:-

- For solid shaft: $T = \frac{\pi}{16}\tau d^3$ --- (I)
- For hollow shaft: $T = \frac{\pi}{16}\tau\left[\frac{d_o^4 - d_i^4}{d_o}\right]$ -- (II)

$\because$ Both shafts transmit the same power & torque, so:

$$\frac{\pi}{16}\tau\left[\frac{d_o^4 - d_i^4}{d_o}\right] = \frac{\pi}{16}\tau d^3 \Rightarrow \boxed{\frac{d_o^4 - d_i^4}{d_o} = d^3}$$

$$\Rightarrow \boxed{d_o^3(1 - k^4) = d^3}, \text{ where, } \boxed{k = \frac{d_i}{d_o}}$$

Q: If solid shaft dia. is 20mm, then find the equivalent hollow shaft whose inside dia. is 20mm.

Ans- Given, $d = 20mm = d_i$; $d_o = ?$

$$\therefore \frac{d_o^4 - d_i^4}{d_o} = d^3 \Rightarrow d_o = 24.414 \text{ mm}$$

- $V_h = \left[\frac{\pi d_o^2}{4} - \frac{\pi d_i^2}{4}\right] \times 1m = 153.972 \times 10^3 \text{ mm}^3$
- $V_s = \frac{\pi}{4}d^2 \times 1m = 314.159 \times 10^3 \text{ mm}^3$

$\therefore V_s - V_h = 160.187 \text{ mm}^3$

N.B: In case of belt drives, the torque (T) is given by:

$$\boxed{T = (T_1 - T_2)R}$$

where, T_1 = tens^n in the tight side.
T_2 = tens^n in the slack side.
R = Pulley radius.

=> Shafts subjected to bending moment only:-

- For solid shafts:

$$\boxed{I = \frac{\pi}{64} d^4} \quad \& \quad \boxed{y = \frac{d}{2}}$$

$$\therefore \frac{M}{I} = \frac{\sigma_b}{y} \Rightarrow \boxed{M = \frac{\pi}{32} \sigma_b d^3}$$

- For hollow shafts:

$$\boxed{I = \frac{\pi}{64} [d_o^4 - d_i^4]} \quad \& \quad \boxed{y = \frac{d_o}{2}}$$

$$\therefore \frac{M}{I} = \frac{\sigma_b}{y} \Rightarrow \boxed{M = \frac{\pi}{32} \sigma_b \left[\frac{d_o^4 - d_i^4}{d_o}\right]}$$

=> Shafts subjected to combined bending & twisting moment:-

- Max. shear stress theory or Guest's theory is used for ductile materials such as mild steel. (IMP)
- Max. normal stress theory or Rankine theory is used for brittle materials such as cast Fe. (IMP)

- For solid shaft:

-> (a) $T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \tau_{(max)} d^3$

-> (b) $M_e = \frac{M + \sqrt{M^2 + T^2}}{2} = \frac{\pi}{32} \sigma_{b(max)} d^3 \Rightarrow \boxed{M_e = \frac{M + T_e}{2}}$

- For hollow shaft:

-> (a) $T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \tau \left[\frac{d_o^4 - d_i^4}{d_o}\right]$ (Max)

-> (b) $M_e = \frac{M + \sqrt{M^2 + T^2}}{2} = \frac{\pi}{32} \sigma_b \left[\frac{d_o^4 - d_i^4}{d_o}\right]$

where, T_e = Equivalent twisting moment

M_e = Equivalent Bending Moment

Mod-4

// → Bearing:- There are 2 main classificat^{n}s of bearing, including:

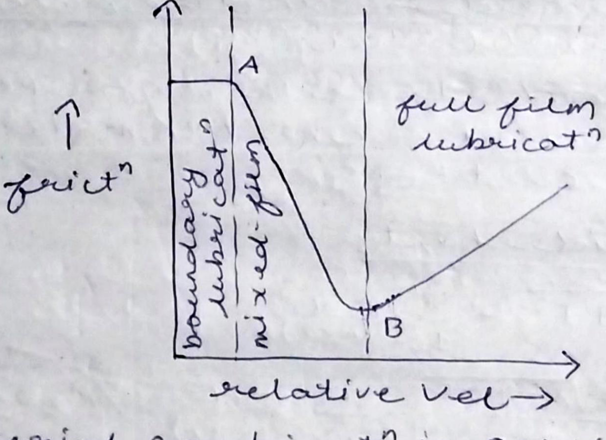
- Plain bearing
- Rolling contact bearing

// → Design of Plain Bearing:-

The 3 general types of lubricat^n are:

- Full film lubricat^n
- Mixed film lubricat^n
- Boundary lubricat^n

Full film lubricat^n can be hydrostatic, hydrodynamic, or elastohydrodynamic, whereas, boundary lubricat^n describes a situat^n where the bearing surfaces physically contact due to reas^{n}s such as geometry, surface roughness, excessive load, or insufficient lubricant, leading to the possibility of adhesive or abrasive wear.



// → Material combinat^n in sliding contact bearing:-

Some properties sought in a bearing material are relative softness, reas^{able} strength, machinability (to maintain tolerances, lubricity, temp., & corros^n resistance, & in some cases, porosity (to absorb lubricant).

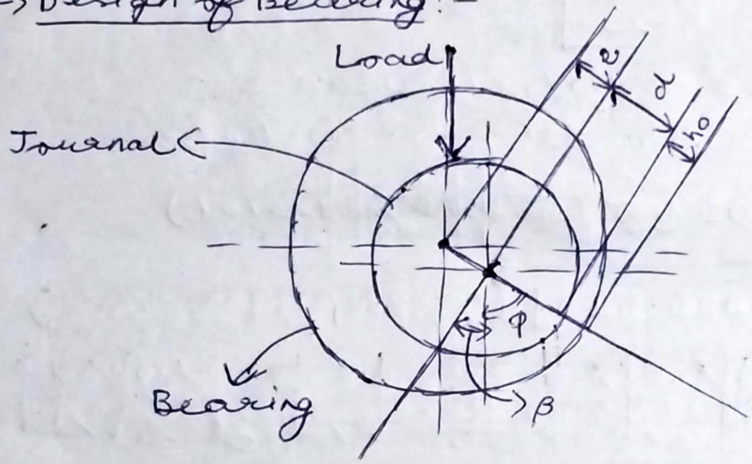
(IMP) A bearing material should be less than 1/3rd as hard as the material running

against it in order to provide embedability of abrasive particles.

// → **Babbits :-**

shafts for babbit bearings should have a min. hardness of 150 – 200HB & a ground surface finish of $R_a = 0.25$ to 0.30 µm.

// → **Design of Bearing :-**



Let, F = Radial load in Newton.

L = Bearing length in metre.

D = Bearing dia. in metre.

p = Pressure in N/m^2 i.e. Pa

v = sliding vel. in m/s

C_d = Diametral clearance

d = Journal or shaft dia.

* $\boxed{p = \dfrac{F}{LD}}$ * $\boxed{v = \text{RPS} \times \text{Bearing circumference}}$

* $\boxed{C_d = D - d}$

* $\dfrac{C_d}{D}$ = Diametral clearance Ratio

$$\therefore \frac{C_d}{D} = \begin{cases} 1\text{ mm/metre (plastic bearing)} \\ 0.001\text{ mm/metre (default value)} \\ 1.5\text{ mm/metre (hard bearing)} \end{cases}$$

* $$\frac{L}{D} = \begin{cases} < 1 \text{ (Short Bearing)} \\ = 1 \text{ (Square Bearing)} \\ > 1 \text{ (Long Bearing)} \end{cases}$$

N.B: If there's a problem of shaft deflect^n, then short bearing will be used.

- e = Eccentricity = Radial dist. between bearing centre & journal's displaced centre under load.

- ε = Eccentricity ratio

$$\Rightarrow \boxed{\varepsilon = \frac{e}{C_r} = \frac{C_r - h_0}{C_r}}$$

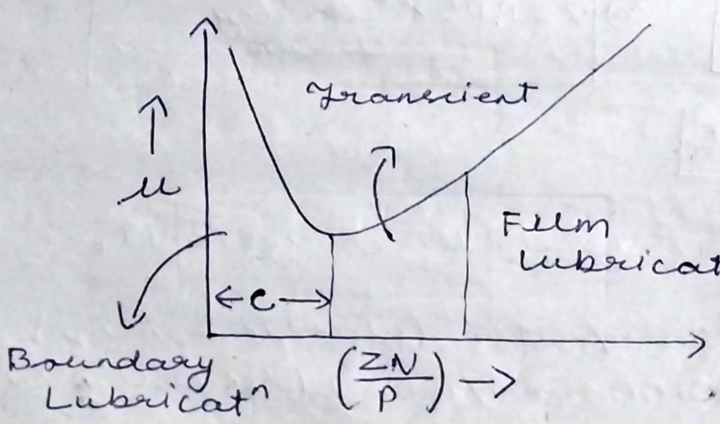
where, C_r = Radial clearance
h_0 = Min. oil film thickness

$$\therefore h_0 = \begin{cases} > 0.02\text{ mm (Babbit material)} \\ \in [0.025, 0.15] \text{ (Large power m/cery} \end{cases}$$

~~$h_0 = \frac{\phi ZN}{p}$ where, $\frac{ZN}{p}$ = C = Bearing Modulus~~

- $$\boxed{h_0 = \frac{\phi ZN}{p}}$$

where, $\frac{ZN}{p}$ = C = Bearing Modulus
Z = Abs. lubricant viscosity
N = Shaft RPM or Shaft rotat^n
p = Pressure or Bearing Pressure

- 

So, for normal loading condit^{n}s:

$$\boxed{\frac{ZN}{p} \geq 3C}$$

But for heavy, impact, or fluctuating loading

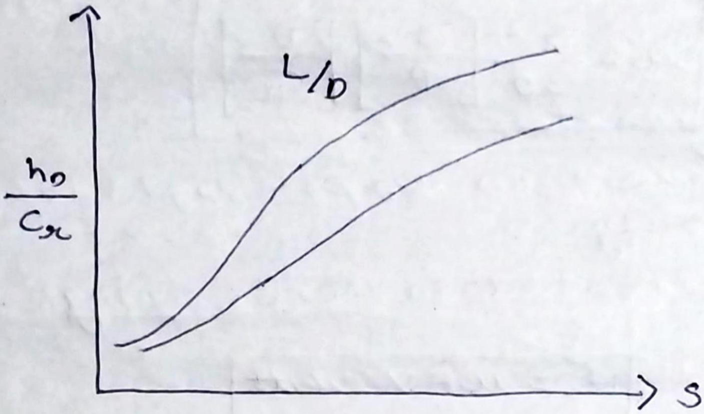
condit^n s:

$$\boxed{\frac{ZN}{p} = 15C}$$

⊛ Sommerfeld No:

$$\boxed{S = \frac{Zn'}{p}\left[\frac{D}{C_d}\right]^2}$$

where, $n' = RPS$
p = Bearing Pressure



⊛ Min. Operating Pressure:

$$\boxed{P_{min} = \frac{ZN}{4.75 \times 10^6}\left[\frac{D}{C_d}\right]^2\left[\frac{L}{D+L}\right] \ MN/m^2}$$

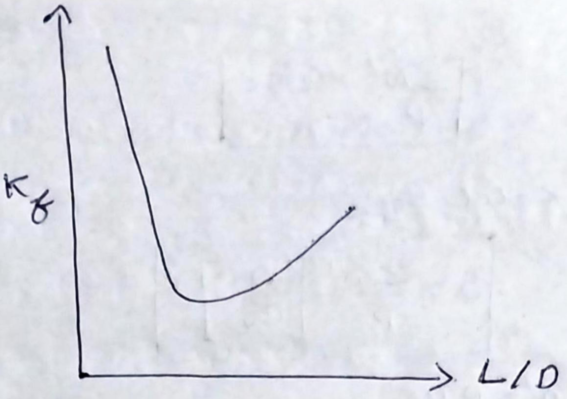
Here, $\boxed{h_o = \frac{C_d}{4}}$ (Assumpt^n)

⊛ ~~Also~~ $\boxed{\mu = \frac{0.326}{10^6}\left[\frac{ZN}{p}\right]\left[\frac{D}{C_d}\right] + K_f}$

where, K_f = Correct^n factor for end leakage.

$\therefore K_f = 0.002$, if $\frac{L}{D} \in [0.75, 2.8]$

N.B: (K_f) factor isn't considered for hydrostatic bearing & pressure fed hydrodynamic bearing. In such cases, $\mu = 0.001$.



* **Petroff's Equn:**

$$\boxed{\mu = \frac{\pi^2}{30}\left[\frac{ZN}{P}\right]\left[\frac{D}{C_d}\right]}$$

The above equn. is for light loading condit's only.

* **Heat generated in the Bearing:**

$$\boxed{H_g = \mu F V \text{ Watt}}$$

where, F = Radial Load

V = Sliding vel.

N.B: The general operating temp. for bearing is 50-60°C.

* **Heat dissipated by the Bearing:**

$$\boxed{H_d = K_d DL(t_b - t_a)}$$

where, $\boxed{t_b - t_a = \frac{t_o - t_a}{2}}$

$$\therefore \boxed{H_d = K_d DL \frac{(t_o - t_a)}{2}} \text{ Watt}$$

where, K_d = Energy dissipat^n coeff in $Wm^{-2}°C$

t_o = Operating temp = 70-82°C

t_b = Bearing temp.

t_a = Atm. or Ambient temp.

1=> **Design procedure:-**

(a) Lubricant select^n (b) Bearing Load.

- $\frac{L}{D}$ ratio
- clearance
- Bearing Pressure
- $\frac{C_d}{D}$
- $\frac{ZN}{p}$
- μ
- H_g
- H_d

Q: A journal bearing is proposed for a centrifugal pump. The dia. of journal is 0.15m & the load on it is 40kN, & the speed is 900RPM. Design the bearing.

Ans- Given, $D = 0.15m$; $F = 40kN$; $N = 900RPM$

- Let's assume $\left(\frac{L}{D}\right)$ ratio for centrifugal pump as 1.6.

[This (L/D) ratio for centrifugal pump generally varies from 1 to 2]

- $L = 1.6D = 1.6 \times 0.15 = 0.24m$

- $p = \frac{F}{LD} = \frac{40 \times 10^3}{0.24 \times 0.15} = 1.11 \text{ MPa}$

- The permissible bearing pressure for pump lies from 0.7 to 1.4 MPa. ∵ 1.11 MPa is within the permissible limit, so the bearing pressure is acceptable.

- $Z = 18$ centipoise $= 18 \times 10^{-2} \times 0.1 \text{ Nsm}^{-2}$
  $= 0.018 \text{ Nsm}^{-2}$ (Assuming operating temp. as 55°C)

- $\frac{ZN}{p} = \frac{0.018 \times 900}{1.11} = 14.595 \times 10^{-6}$ sec

- For pump, the operating $\left(\frac{ZN}{p}\right)$ is around 28.50×10^{-6} sec

- $\frac{ZN}{p} \geq 3C \Rightarrow C = \frac{28.50 \times 10^{-6}}{3}$

  $\Rightarrow C = 9.5 \times 10^{-6}$ sec

- $\mu = \frac{0.326}{10^6}\left[\frac{ZN}{p}\right]\left[\frac{D}{C_d}\right] + K_f$

From table, $\frac{C_d}{D} = 0.001$

$K_f = 0.0024$ for $\frac{L}{D} = 1.6$

$\therefore \mu = 0.0024$

- $H_g = \mu F V = \mu F \times \frac{\pi D N}{60} = 678.584$ watt
- $H_d = K_d D L \frac{(t_o - t_a)}{2}$ watt

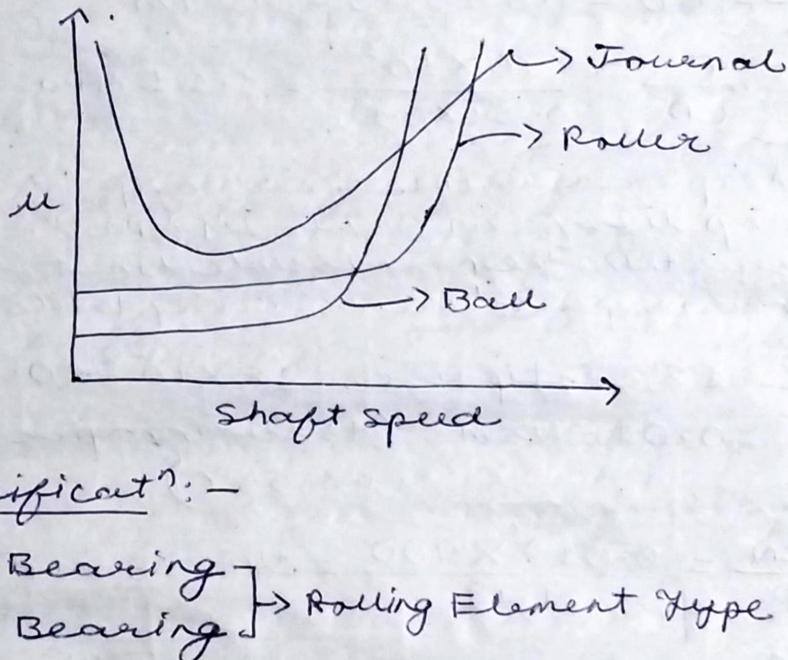
From table, $K_d = 1228.13\ W m^{-2} °C^{-1}$

& let, $t_a = 25°C$

$\therefore H_d = 663.1902$ watt

$\because$ The heat generated & heat dissipated are nearly equal or are very close to each other, so the bearing design is safe.

II → Rolling contact Bearing :-



I → Classificat^n :-

- Ball Bearing
- Roller Bearing

} → Rolling Element Type

- Radial Bearing
- Lular Bearing
- Thrust Bearing

} → Load Direct^n

- Light Bearing
- Medium Bearing
- Heavy Bearing

} → Load type

- Single Row Bearing
- Double Row Bearing
- Multi Row Bearing

} → No. of rows of rolling elements

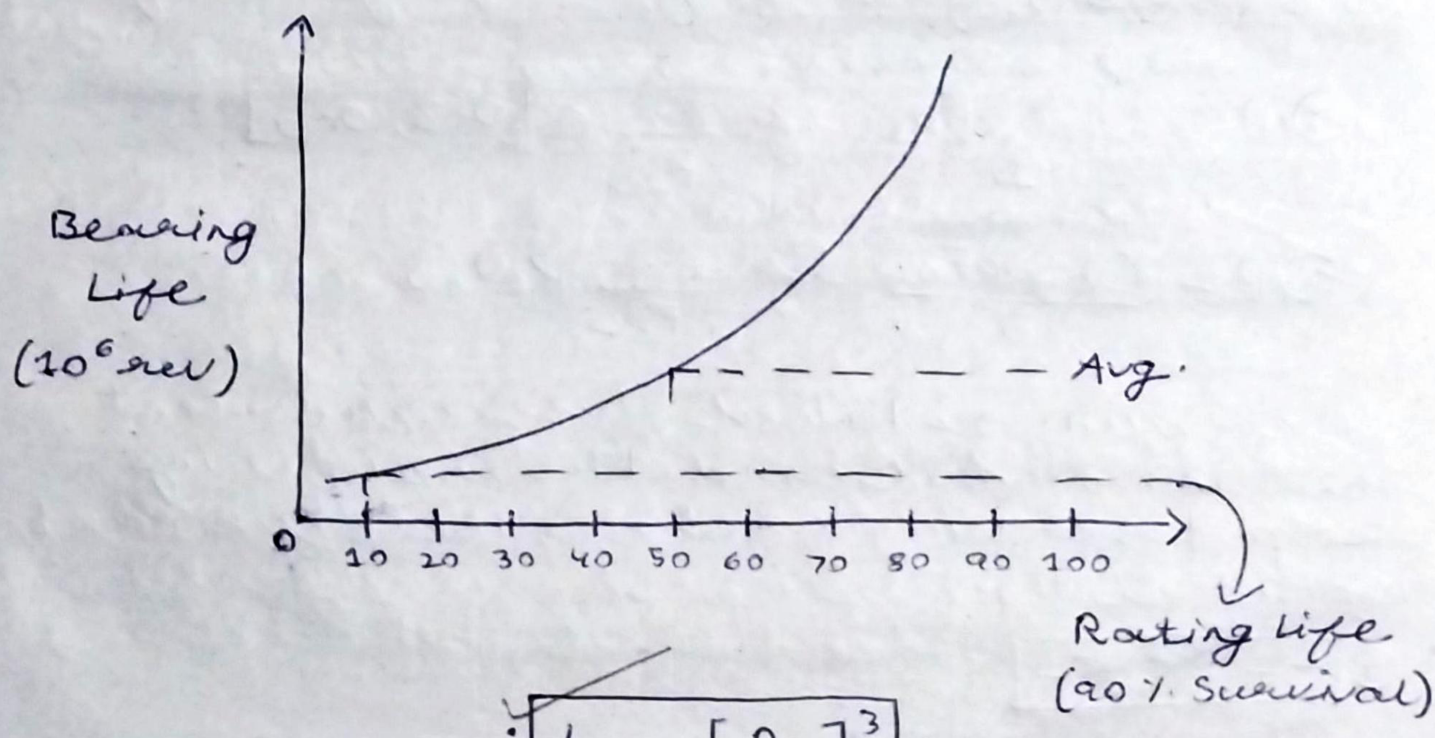
⇒ Bearing Specs :-

- Extra light load = 100
- Light load = 200
- Medium load = 300
- Heavy load = 400
- Very heavy load = 500

$$\boxed{6203-C-2HRS}$$

Multiplying (03) with (5) will give bore dia.

⇒ Bearing Life :-



$$\therefore \boxed{\frac{L_1}{L_2} = \left[\frac{P_2}{P_1}\right]^3}$$

- If $L_1 = 10^6$ rev, then $P_2 = C =$ Basic load rating.

- Here, (C) is defined as the radial load that a ball bearing can withstand for 1 million rev. of inner ring with 10% failure. The (c) is also called specific dynamic capacity or basic dynamic capacity or dynamic load rating.

- Rating Life $(L) = \left[\frac{C}{P}\right]^3$ million rev.

$\Rightarrow L = \left[\frac{C}{P}\right]^3 \times \frac{10^6}{60n}$ hrs

1 $\Rightarrow$ Dynamically Loaded Bearing:—

$$\boxed{P = XVF_r + YF_a = VF_r}$$

where, V = Rotat^nal factor

- $V = \begin{cases} 1, & \text{for all types of bearing if inner race is rotating.} \\ 1.2, & \text{for all types of bearing if inner race is stat}^{n}\text{ary, except self-aligning bearing.} \end{cases}$

- $X = \begin{cases} 1, & \text{for general bearing} \\ 0.5, & \text{for cular \& self aligning bearing.} \end{cases}$

- $Y = \begin{cases} 1.5, & \text{for general bearing} \\ 1, & \text{for cular bearing} \\ 2.5, & \text{for self-aligning bearing} \end{cases}$

- $P = \frac{F_r + 3F_a}{2}$ if $\frac{F_a}{F_r} \in [0.3, 0.5]$

- $P = \frac{F_r + 4F_a}{3}$ if $\frac{F_a}{F_r} > 0.5$

The eqn. obtained gives equivalent load in normal condit^n. Considering temp. & service factor, the eqn can be modified to:

- $\boxed{F = PK_aK_T}$

where, K_a = Applicat^n or service factor
K_T = Temp. factor

In the eqn, the rotat^nal speed isn't considered $\because$ speed of rotat^n plays a

significant role in fatigue life, ∴ the eqn. has to be modified.

$$\therefore C_R = F \times \left[\frac{L_d}{L_c}\right]^{1/3} \times \left[\frac{n_d}{n_c}\right]^{1/3}$$

$$\Rightarrow \boxed{C_R = F \times \sqrt[3]{\frac{L_d}{L_c}} \times \sqrt[3]{\frac{n_d}{n_c}}}$$

$$\Rightarrow \boxed{C_R = F \times \sqrt[3]{\frac{L_d n_d}{L_c n_c}}}$$

where, C_R = Req. radial factor

L_d = Desired bearing life in hrs.

L_c = Catalogue bearing life in hrs.

n_d = Desired rotat'al speed of bearing in rev./min.

n_c = Catalogue bearing speed in rev/min.

$$\therefore \boxed{C_R = F \times K_L \times K_s}$$

where, K_L = Life factor

K_s = Speed factor

$$\therefore \boxed{C_R = P \times K_a \times K_T \times K_L \times K_s}$$

Q: A bearing is req. to carry 4500kN stat'ary radial load. The shaft rotates at 1000 rpm & life desired is 30,000 hrs. Running condit'ns are steady. No shock loading. Select a bearing.

Ans- $K_a = 1$ (No shock loading)

$K_T = 1$ (No high temp. use)

$$\therefore K_L = \sqrt[3]{\frac{L_d}{L_c}} = \sqrt[3]{\frac{30,000}{10,000}} = 1.44$$

$$\therefore K_s = \sqrt[3]{\frac{n_d}{n_c}} = \sqrt[3]{\frac{1000}{500}} = 1.26$$

$$\therefore C_R = P \times K_a \times K_T \times K_L \times K_s = 81,77,042.668\ N$$

$$= 8.177\ MN$$

Q: The spindle of a wood working m/c revolves at 1000 rpm & it's to be mounted on 2 single row radial ball bearings. Bearing 1 is subjected to a radial load of 2250N & a thrust load of 1900N

Bearing 2 is subjected to a radial load of 2250 N only. The m/c is to be used approx. 8 hrs per day & avg. service life of 10 yrs is desired. If the spindle dia isn't to exceed 100 mm, select the suitable bearing.

Ans:- $\frac{F_a}{F_r} = \frac{1900}{2250} = 0.84 > 0.5$

$\therefore P = \frac{F_r + 4F_a}{3} = 3283.33\ N$

$\therefore K_a = 2$ (shock load)

$\therefore K_T = 1$ (Room temp.)

$\therefore$ No. of working days / year = 365 days

$\therefore$ Desired bearing life $= 8 \times 365 \times 10$

$= 29200$ hrs

$\therefore K_L = \sqrt[3]{\frac{L_d}{L_c}} = 1.43 = \sqrt[3]{\frac{29200}{10,000}}$

$\therefore K_S = \sqrt[3]{\frac{N_d}{N_c}} = \sqrt[3]{\frac{1000}{500}} = 1.26$

$\therefore C_R = 3283.33 \times 2 \times 1 \times 1.43 \times 1.26$

$= 11831.81\ N = 11.83\ kN$ (Bearing 1)

$\therefore C_R = 2250 \times 2 \times 1 \times 1.43 \times 1.26$

$= 8108.1\ N = 8.1\ kN$ (Bearing 2)

II→ Thrust Bearing :-

It's a plain bearing which's used for taking only axial load.

It's used in m/cs & equipments to support & guide shaft or member.

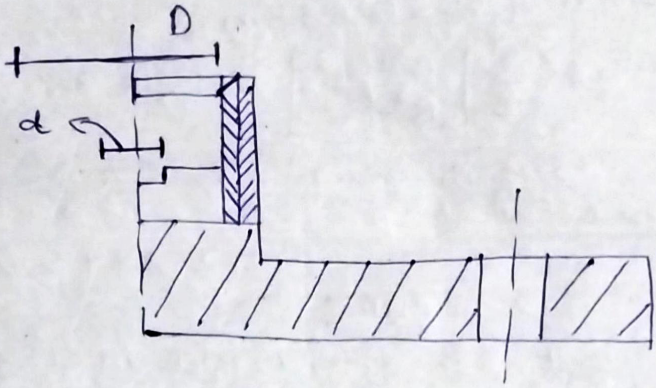
1) classificat^n of Thrust Bearing:-

(a) Step bearing for vertical load.
collar bearing for horizontal load.

(b) Marginally & hydrodynamically lubricated cast Fe, Bronze, & steel with babbit lining bearings.

N.B: The sliding vel. is max. at the bearing periphery & min at the centre. To compensate this differential wear, a step or groove is provided at the bearing centre. Other method is to provide alternate layers

of material at the bottom of the bearing like steel or bronze.



Cross-Sectⁿ of Thrust Bearing

/=> Design of Thrust Bearing:-

* F_a = Axial Load $= PA = P \times \frac{\pi}{4} D^2$

* If uniform pressure distributⁿ is assumed, the frictⁿal torsⁿal moment at the mean dia. will be:

$$\boxed{F_a = P \times \frac{\pi}{4}(D^2 - d^2)}$$

* $T_f = \frac{\mu F_a D_{mean}}{2} = \mu F_a R_{mean}$

where, $R_{mean} = \frac{2}{3}\left[\frac{R^3 - r^3}{R^2 - r^2}\right]$

where, $R = D/2$ & $r = d/2$

$$\therefore \boxed{T_f = \mu F_a \frac{2}{3}\left[\frac{R^3 - r^3}{R^2 - r^2}\right]}$$

* Power lost in frictⁿ $= T_f \omega = T_f \times \frac{\pi N}{30}$ watt

Q: Calculate the dimensⁿs of a bearing disc of a step bearing. Load supported is 20kN. Also, find the power lost in frictⁿ if the avg. speed of shaft is 750 rpm. Take central hole dia as 1/4 th of the outer dia. Assume pressure to be 0.4 MN/m² & coeff. of frictⁿ as 0.015.

Ans- Given, $D = ?$; $d = D/4$; $F_a = 20kN$; $N =$ 750 rpm

* $F_a = P \times \frac{\pi}{4}(D^2 - d^2) = P \times \frac{\pi}{4}\left[D^2 - \left(\frac{D}{4}\right)^2\right]$

$\Rightarrow 20\times10^3 = 0.4\times10^6 \times \frac{\pi}{4}\left[D^2-\left(\frac{D}{4}\right)^2\right]$

$\Rightarrow D = 0.26\,m$ & $d = 0.065\,m$

(b) $T_f = \mu F_a \times \frac{2}{3}\left[\frac{R^3-r^3}{R^2-r^2}\right] \Rightarrow T_f = 27.3\,Nm$

(c) Power loss in frict^n $= T_f \times \frac{\pi N}{30}$

$= 2.144\,kW$